Modulational Instability and Freak Waves

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General Overview

- What is a freak wave?
- How and why is it generated?
- Prediction and probability of occurrence?
- Simulation?
- What is the connection with Solitary waves and Solitons?
- What is the association between freak waves and modulational instability?

Definition of Freak waves

Haver 2000

A freak wave event is an event that represents an outlier when seen in view of population of events, generated by a piecewise stationary and homogeneous second order model of the surface process. (non-Gaussian random Field)[8]

Gaussian Approach

Freak waves are extreme waves of a population of waves with corresponding crest height H_F satisfying the inequality $H_F > 2H_S$, where H_S is the significant height of the population with respect to the Rayleigh distribution [7]

The Probability of occurrence

1. Strong dispersion of the water waves implies that each individual sine wave travels with a frequency dependent velocity

2. Due to nonlinearity of the water waves each individual sine waves interact each to other generating new spectral components

The wave field gives rise to an irregular sea surface that is constantly changing with time

Gaussian random Field with probability density distribution

$$f(\eta) = \frac{1}{\sqrt{2\pi}\sigma} \exp(\frac{-\eta^2}{2\sigma^2}) \tag{1}$$

η the sea level displacement with zero mean level
σ^2 is computed from frequency spectrum $S(\omega)$

$$\sigma^2 =< \eta^2 >= \int_0^\infty S(\omega)d\omega \tag{2}$$

Further Assumptions

1. The wind spectrum is narrow

2. Significant height
$H_s = (3\sqrt{2\pi}[error function](\sqrt{\ln(3)} + 2\sqrt{2\ln(3)})\sigma \cong 4\sigma$

Rayleigh Wave Height Distribution

$$P(H) = \exp(\frac{-H^2}{8\sigma^2}) \qquad (3)$$

Conclusion

Probability of Freak Wave event < 0.000336

Limit Theorem of Linear Approximation

Maximum Freak way event $\rightarrow H_F = 3H_S$

BUT In practice have been scanned Freak waves with

$H_F = 4H_S$

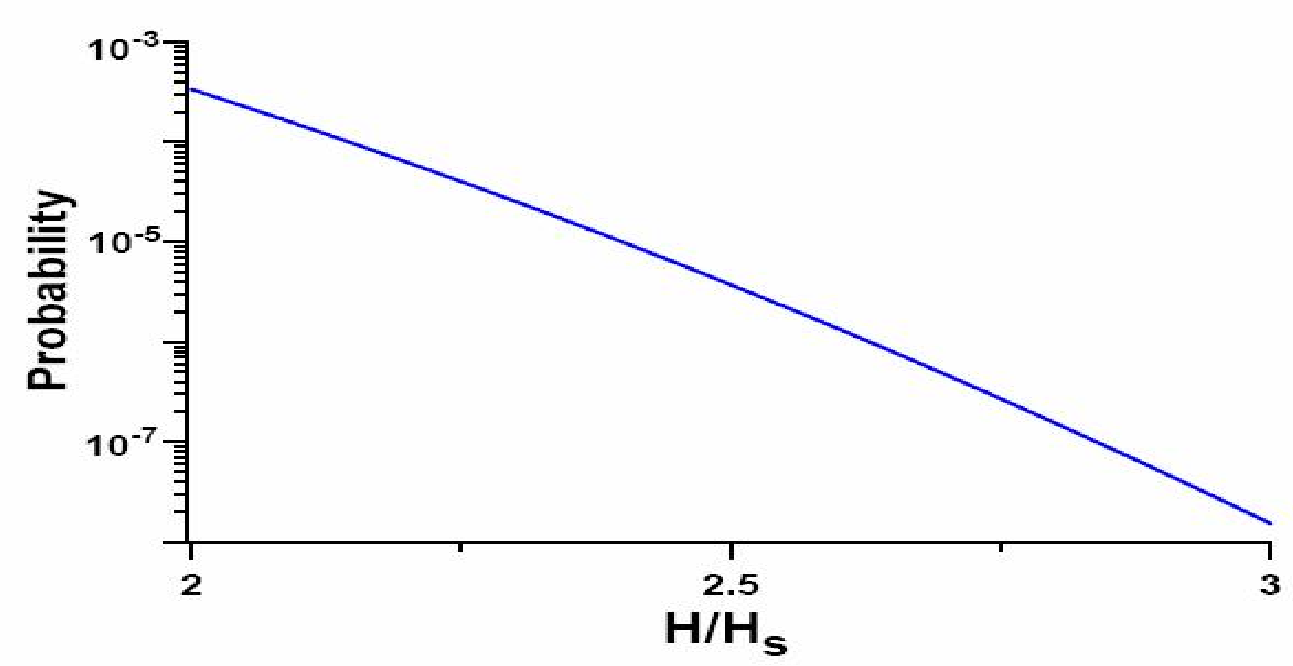


Figure 3.1. Probability of the freak wave formation

HAVER's Definition ⟶ Statistics of large deviations ⟶ more realistic Probabilities

True Probability $\geq$ Linear Theory Probability

Freak Waves' Properties

- Freak Waves are essentially non linear objects
- Very steep. In the last stage of evolution steepness becomes infinite, thus forming a wall of water
- They are single events
- Small characteristic life (10 wave periods)
- Almost instant appearance through relatively calm sea

Theorem 0.0.1 *Progressive waves of finite amplitude on deep water (Stokes Waves) are unstable [1]*

NOTES

- Benjamin-Feir instability or Modulational instability
- Deep water corresponds to infinite depth water
- Naturally non linear objects associated with instability

Assumptions and general idea

(1) Basic Wave train:

amplitude$= \alpha$

argument$= \zeta = kx - \omega t$, harmonics: $2\zeta, \ldots$

advance in horizontal x-direction.

phase velocity $c = \omega / k$

(2) Disturbance waves:

Pair of progressive waves with:

side band frequencies, wave numbers adjacent to ω, k and

$$\zeta_1 = k(1+\kappa)x - \omega(1+\delta)t - \gamma_1 \quad (4)$$

$$\zeta_2 = k(1-\kappa)x - \omega(1-\delta)t - \gamma_2 \quad (5)$$

κ, δ, small fractions

Amplitudes: $\epsilon_1, \epsilon_2 << \alpha$

The non-linear interaction of the above $\longrightarrow$ components with arguments:

$$2\zeta - \zeta_1 = \zeta_2 + (\gamma_1 + \gamma_2) \tag{6}$$
$$2\zeta - \zeta_2 = \zeta_1 + (\gamma_1 + \gamma_2) \tag{7}$$

with amplitudes proportional to: $\alpha^2\epsilon_1, \alpha^2\epsilon_2$.

Assumption: If

$$\theta = \gamma_1 + \gamma_2 \rightarrow constant \tag{8}$$

then:

Each mode suffers a synchronous forcing action proportional to the amplitude of the other, so that the two can grow mutually at an exponential rate

Crucial Task

Given α, k, ω of the basic wave train, property (5) can hold for some non-zero κ, δ.

Stability Analysis

Boundary Problem

Equation of free surface:

$$y = \eta(x,t) \tag{9}$$

η is the elevation of the surface above its mean level $y = 0$

Conditions

Velocity Potential:

$$\bigtriangledown^2\phi = \phi_{xx} + \phi_{yy} = 0 \tag{10}$$

No motion in infinite depths:

$$\bigtriangledown\phi \rightarrow 0, y \rightarrow -\infty \tag{11}$$

Kinematical boundary condition:

$$D(\eta - y)/Dt = \eta_t + \eta_x[\phi_x]_{y=\eta} - [\phi_y]_{y=\eta} = 0 \tag{12}$$

Condition of constant pressure(Surface tension assumed absent)

$$g\eta + [\phi_t]_{y=\eta} + \frac{1}{2}[\phi_x^2 + \phi_y^2]_{y=\eta} = 0 \tag{13}$$

Existence of periodic solutions of the form:
$\eta = H(x-ct), \phi = \Phi(x-ct, y)$, Levi-Civita (1925)

Approximation up to α^2 terms because:
The affect of a nearby train wave to the original one in the phase velocity will be of second order the amplitude responsible for it

Analytical Form of Solutions

$$\eta = H = \alpha \cos(\zeta) + \frac{1}{2} k\alpha^2 \cos(2\zeta) \quad (14)$$

$$\phi = \Phi = \omega k^{-1} \alpha e^{ky} \sin(\zeta) \quad (15)$$

(16)

where

$$\omega^2 = gk(1 + k^2\alpha^2) \quad (17)$$

Sufficient accuracy if $k\alpha$ is small

Perturbation Equations

Let:

$$\phi = \Phi + \epsilon\tilde{\phi}, \eta = H + \epsilon\tilde{\eta} \tag{18}$$

(10) is linear hence:

$$\bigtriangledown^2\tilde{\phi} = \tilde{\phi_{xx}} + \tilde{\phi_{yy}} = 0 \tag{19}$$

$$\bigtriangledown\tilde{\phi} \to 0, y \to -\infty \tag{20}$$

Substitution to (12)(13) and linearization in ϵ gives:

$$\tilde{\eta}_t + \tilde{\eta}_x[\Phi_x]_{y=H} + \tilde{\eta}[-\Phi_{yy} + H_x\Phi_{xy}]_{y=H} + [-\tilde{\phi}_y + H_x\tilde{\phi}_x]_{y=H} = 0 \quad (21)$$

$$g\tilde{\eta} + \tilde{\eta}[\Phi_x\Phi_{xy} + \Phi_y\Phi_{yy} + \Phi_{ty}]_{y=H} + [\tilde{\phi}_t + \Phi_x\tilde{\phi}_x + \Phi_y\tilde{\phi}_y]_{y=H} = 0 \quad (22)$$

1) Simplification of (21)(22) up to terms α^2.

2)Analytical continuation of $\tilde{\phi}$ in a neighborhood about y=H.

Taylor expansion gives

$$\tilde{\eta}_t - [\tilde{\phi}_y]_{y=0} = \alpha[k\omega\sin(\zeta)\tilde{\eta} - \omega\cos(\zeta)\tilde{\eta}_x + (\cos(\zeta)\tilde{\phi}_{yy} + k\sin(\zeta)\tilde{\phi}_x)_{y=0}]$$
$$+ \tfrac{1}{2}\alpha^2[2k^2\omega\sin(2\zeta)\tilde{\eta} - k\omega(1+\cos(2\zeta))\tilde{\eta}_x$$
$$+\{k\sin(2\zeta)(2k\tilde{\phi}_x + \tilde{\phi}_{xy}) + k\cos(2\zeta)\tilde{\phi}_{yy} + \frac{1}{2}(1+\cos(2\zeta))\tilde{\phi}_{yyy}\}_{y=0}] \quad (23)$$

$$g\tilde{\eta} + (\tilde{\phi}_t)_{y=0} = \alpha[\omega^2\cos(2\zeta)\tilde{\eta} - (\omega\cos(\zeta)\tilde{\phi}_x + \omega\sin(\zeta)\tilde{\phi}_y + \cos(\zeta)\tilde{\phi}_{yt})_{y=0}]$$
$$- \tfrac{1}{2}\alpha^2[k\omega^2(1-\cos(2\zeta))\tilde{\eta} + \{\omega\sin(2\zeta)(k\tilde{\phi}_y + \tilde{\phi}_{yy})$$
$$+(1+\cos(2\zeta))(k\omega\tilde{\phi}_x + \omega\tilde{\phi}_{xy} + \frac{1}{2}\tilde{\phi}_{yyt}) + k\cos(2\zeta)\tilde{\phi}_{yt}\}_{y=0}] \quad (24)$$

Assumed form of solution

The main assumption enters:

$$\tilde{\eta} = \tilde{\eta}_1 + \tilde{\eta}_2 \tag{25}$$

The two components have the following form:

$$\tilde{\eta}_i = \epsilon_i \cos(\zeta_i) + k\alpha\epsilon_i\{A_i \cos(\zeta + \zeta_i) + B_i \cos(\zeta - \zeta_i)\} + O(k^2\alpha^2\epsilon_i) \tag{26}$$

1. A_i, B_i are of $O(1)$
2. $O(k^2\alpha^2\epsilon_i)$ have arguments $2\zeta + \zeta_i$ and are of no matter in the stability analysis
3. Terms with arguments $2\zeta - \zeta_i$ play the crucial role

ϕ form of solution and dispersion

Under the same assumptions we have:
$\tilde{\phi} = \tilde{\phi}_1 + \tilde{\phi}_2$

where

$$\tilde{\phi}_i = k_i^{-1} e^{k_i y} \{\epsilon_i(\omega_i' L_i + \dot{\gamma}_i M_i)\sin(\zeta_i) + \dot{\epsilon}_i N_i \cos(\zeta_i)\}$$
$$+\omega\alpha\epsilon_i \{C_i e^{|k+k_i|y} \sin(\zeta+\zeta_i) + D_i e^{|k-k_i|} y \sin(\zeta-\zeta_i)\} \quad (27)$$

where $k_i = k(1 \pm \kappa)$ and $\omega_i' = \omega(1 \pm \delta)$

Let $\delta = (1/2)\kappa$

Then:

k_i, ω_i satisfy the dispersion relation to a first approximation since:

Dispersion relation= $\omega_i^2 = gk_i$

even if γ_i are both constant.

Evaluation of coefficients $L_i = 1 + O(k^2\alpha^2)$
M_i, N_i, C_i, D_i are of $O(1)$

Notions

-The boundary conditions (23)(24) are to be satisfied over a continuous and unbounded range of x

- If all terms are reduced to simple harmonic components, then each set of components must satisfy the above condition independently

Determination of $\epsilon_i(t), \gamma_i(t)$

1st Step coefficients of the terms proportional to $\alpha\epsilon_i$ in (25)

Separation of components in (19)(20) with arguments $\zeta \pm \zeta_i$, substitution of the zeroth approximation:

$\tilde{\eta}_i = \epsilon_i \cos(\zeta_i), \tilde{\phi}_i = k_i^{-1}\omega_i\epsilon_i e^{k_i y}\sin(\zeta_i), L_i = 1$

and trigonometric reductions give:

The coefficients in the left side and the terms of required order in the right hand side

Separation of components at wave numbers $k \pm k_i$ leads to equations for A_i, C_i and B_i, D_i and finally:

$$A_i = 1, B_i = 0, C_i = 0, D_i = \pm 1 \tag{28}$$

if $O(\delta)$ is neglected ***

2nd Step The terms $O(\omega k^2 \alpha^2 \epsilon_i)$, $O(\omega^2 k \alpha^2 \epsilon_i)$

-Separation of components at wave numbers k_i in (19),(20) with the required approximation

-Lots of reductions...

$(23) \rightarrow$

$$\epsilon_{1,2}\{\omega'_{1,2}(1-L_{1,2})+\dot{\gamma}_{1,2}(1-M_{1,2})\}\sin(\zeta_{1,2})+\dot{\epsilon}_{1,2}(1-N_{1,2})\cos(\zeta_{1,2})$$
$$= \omega k^2 \alpha^2 \{(5/4)\epsilon_{1,2}\sin(\zeta_{1,2}) + (5/8)\epsilon_{2,1}\sin(\zeta_{1,2}+\theta)\} \quad (29)$$

$(24) \rightarrow$

$$\epsilon_{1,2}\{\omega_1'^{-1}(gk_{1,2} - \omega_{1,2}'^2 L_{1,2}) - \dot{\gamma}_{1,2}(1 + M_{1,2})\}\cos(\zeta_{1,2}) + \dot{\epsilon}_{1,2}(1 + N_{1,2})\sin(\zeta_{1,2})$$
$$= -\omega k^2 \alpha^2 \{(3/4)\epsilon_{1,2}\cos(\zeta_{1,2}) + (3/8)\epsilon_{2,1}\cos(\zeta_{1,2}+\theta)\} \quad (30)$$

The differential equations for ϵ_i, γ_i

$$d\epsilon_{1,2}/dt = ((1/2)\omega k^2 \alpha^2 \sin(\theta))\epsilon_{2,1} \tag{31}$$

$$d\gamma_{1,2}/dt = 1/2\{\frac{gk_{1,2}}{\omega' 1,2} - \omega'_{1,2}\} + \omega k^2 \alpha^2 \{1 + (1/2)\frac{\epsilon_{2,1}}{\epsilon_{1,2}} \cos(\theta)\} \tag{32}$$

Demonstration of instability

Integration of (31) gives:

$$\epsilon_{1,2} = \epsilon_{1,2}(0) \cosh((1/2)\omega k^2 \alpha^2 \textstyle\int_0^t \sin(\theta) dt)$$
$$+\epsilon_{2,1}(0) \sinh((1/2)\omega k^2 \alpha^2 \int_0^t \sin(\theta) dt) \tag{33}$$

Let $T = k^2\alpha^2\omega t$ and $a = \frac{k^2\alpha^2-\delta^2}{k^2\alpha^2}$ and multiply by $\epsilon_1\epsilon_2\sin(\theta)$ we obtain:

$$-\epsilon_1\epsilon_2 d(\cos(\theta))/dT = \alpha\epsilon_1\epsilon_2\sin(\theta) + (1/2)(\epsilon_1^2+\epsilon_2^2)\sin(\theta)\cos(\theta) \quad (34)$$

Now (31) gives $d\epsilon_1^2/dT = d\epsilon_2^2/dT = \epsilon_1\epsilon_2\sin(\theta)$ hence (34) is transformed to

$$d(\epsilon_1\epsilon_2\cos(\theta) + \alpha\epsilon_1^2)/dT = 0 \quad (35)$$

and combination of the above gives finally:

$$(d\epsilon_1^2/dT)^2 = (1-a^2)\epsilon_1^4 + 2av\rho\epsilon_1^2 - \rho^2 \quad (36)$$

Let $d\epsilon_1^2/dT = Q$. The two roots have the representation $A \pm B$ where:

$$A = -\frac{av\rho}{1-a^2} \text{ and } B = \frac{\rho(1-a^2+a^2v^2)^{1/2}}{|1-a^2|}$$

Conclusion If $-1 < a < 1$ then one root is positive and any value of ϵ_1^2 greater than this makes Q positive hence, the unbounded growth of ϵ_1^2 with increasing T is possible

Modulational Instability Implies Freak waves ([5])

Scaling of methodology and Results

- Boundary Problem
- $\mathcal{T} = (-1/2) \int_{-\infty}^{\infty} \psi \phi_n dx, U = \int_{-\infty}^{\infty} \eta^2(x,t) dx$ kinetic, potential energy
- Euler equation of hydrodynamics

$$\frac{\partial \eta}{\partial t} = \frac{\delta H}{\delta \psi} \qquad (37)$$

$$\frac{\partial \psi}{\partial t} = -\frac{\delta H}{\delta \eta} \qquad (38)$$

- Conformal mappings and transformations to Z-plane
- Numerical approximation with spectral codes

Numerical results

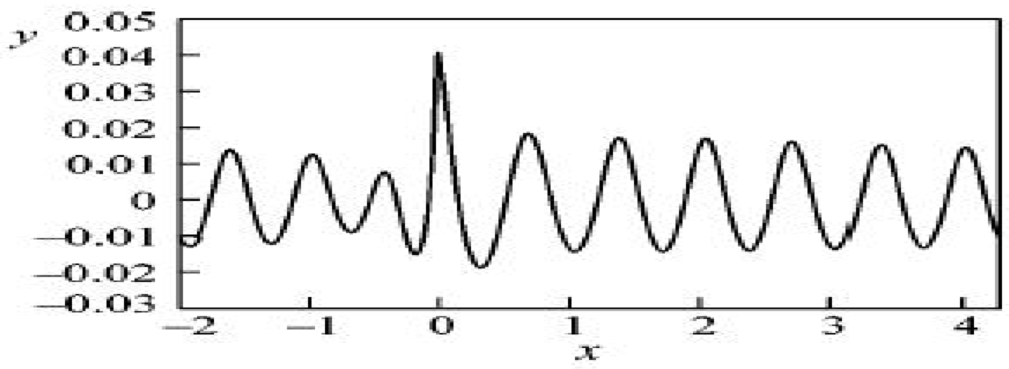


Fig. 2. The shape of the surface at $T = 458.56$.

One can see fast, non-monotonic formation of the freak wave. At this moment, the freak wave is more steep than the Stokes wave of limiting amplitude. The amplitudes of the waves preceding the freak wave are relatively small (three times less). One can see a trough just ahead of the freak wave. This is the so-called hole in the water (marine folklore) that precedes a freak wave

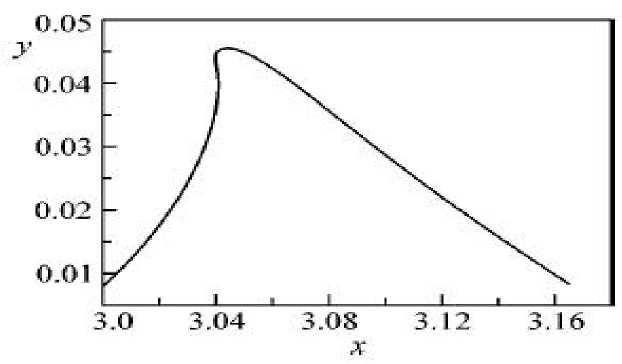


Fig. 4. The shape of the surface near the wave crest at $T = 458.842$.

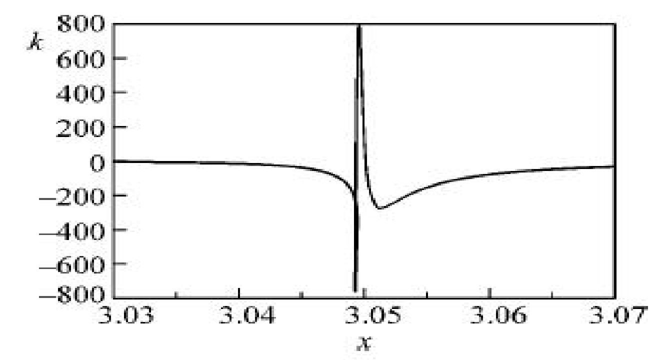


Fig. 5. Curvature (k) of the surface at $T = 458.842$.

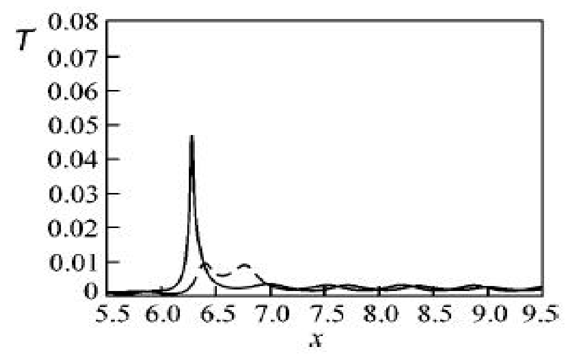


Fig. 6. The density of the kinetic energy just before breaking at $T = 456$ (dashed line) and at the moment of breaking at $T = 458.5$ (solid line).

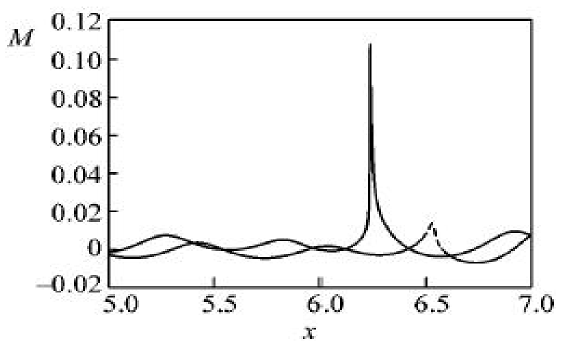


Fig. 7. Distribution of momentum (M) before (dashed line) and after (solid line) breaking.

Explanation of Freak waves

Chaos Theory | Optimization | Envelope Solutions | Large deviations

The envelope equations

NLS describes slowly modulated nonlinear Stokes waves

Freak wave description not sufficient because NLS is derived by expansion of series on powers of $\lambda \simeq (Lk)^{-1}$, k is the wave number and L the length of the modulation but for real freak wave $\lambda \sim 1$ and slow modulation expansion fails

1st Step

Modulational instability of NLS [3,2]

The proof is given for $\psi_0 = C$ The result generalizes with Floquet Spectral Theory

Proof

$$NLS \quad i\psi_y + D\psi + \gamma|\psi|^2\psi = 0, D = a_{ij}\partial_i\partial_j \tag{39}$$

Exact solution

$\psi = \psi_0 e^{i\gamma|\psi_0|^2 t}$

Perturbation equations of amplitude and phase

$$\psi = \psi_0(1+\tilde{\psi})e^{i(\gamma|\psi_0|^2 t+\tilde{\phi})} \quad (40)$$

$$\approx \psi_0(1+\tilde{\psi}+i\tilde{\phi})e^{i\gamma|\psi_0|^2 t} \quad (41)$$

System for determination of stability

$$\tilde{\psi}_t + D\tilde{\phi} = 0 \quad (42)$$

$$\tilde{\phi} - D\tilde{\psi} - 2\gamma|\psi_0|^2\tilde{\psi} = 0 \quad (43)$$

Crucial step: Look for Harmonic perturbations proportional to $e^{ik\cdot x}e^{\sigma t}$. Then:

$$\sigma^2 = 2\gamma|\psi_0|^2 a_{ij}k_ik_j - (a_{ij}k_ik_j)^2 \tag{44}$$

If $\gamma a_{ij}k_ik_j$ is positive and $2|\psi_0|^2 > \gamma^{-1}a_{ij}k_ik_j$ the perturbation amplitude is exponentially amplified

2nd Step

NLS B-F instability implies Freak Wave event

- delicate balance between nonlinearity and wave dispersion
- B-F instability almost robust against a narrow spectrum random field

To be demonstrated

Freak wave event out of a coupled system of NLS equations[4]

Coupled NLS system from Zakharov equation

$$\frac{\partial A}{\partial t}+C_x\frac{\partial A}{\partial x}+C_y\frac{\partial A}{\partial y}-i\alpha\frac{\partial^2 A}{\partial x^2}-i\beta\frac{\partial^2 A}{\partial y^2}-i\gamma\frac{\partial^2 A}{\partial x\partial y}+i(\xi|A^2|A+2\zeta|B^2|A)=0 \quad (45)$$

$$\frac{\partial B}{\partial t}+C_x\frac{\partial B}{\partial x}+C_y\frac{\partial B}{\partial y}-i\alpha\frac{\partial^2 B}{\partial x^2}-i\beta\frac{\partial^2 B}{\partial y^2}-i\gamma\frac{\partial^2 B}{\partial x\partial y}+i(\xi|B^2|B+2\zeta|A^2|B)=0 \quad (46)$$

- x-axis is the middle between the two directions of propagation $k_A=(k_{A,x},k_{A,y})\equiv(k,l), k_B=(k_{B,x},k_{B,y})\equiv(k,-l)$, assume both $k,l>0$
- Dispersion relation $\omega_j=\sqrt{g|k_j|}$
- Propagation implies $\omega_A=\omega_B=\sqrt{g\sqrt{k^2+l^2}}$

Analysis

Multiplication by i to the coupled system gives

$$i\left(\frac{\partial A}{\partial t}+C_x\frac{\partial A}{\partial x}+C_y\frac{\partial A}{\partial y}\right)+\alpha\frac{\partial^2 A}{\partial x^2}+\beta\frac{\partial^2 A}{\partial y^2}+\gamma\frac{\partial^2 A}{\partial x\partial y}-\xi|A^2|A-2\zeta|B^2|A=0 \quad (47)$$

$$i\left(\frac{\partial B}{\partial t}+C_x\frac{\partial B}{\partial x}+C_y\frac{\partial B}{\partial y}\right)+\alpha\frac{\partial^2 B}{\partial x^2}+\beta\frac{\partial^2 B}{\partial y^2}+\gamma\frac{\partial^2 B}{\partial x\partial y}-\xi|B^2|B-2\zeta|A^2|B=0 \quad (48)$$

- A, B are the amplitudes
- $C_x = \omega k/2\kappa^2, C_y = \omega l/2\kappa^2$ group speed
- $\alpha = \omega(2l^2 - k^2)/8\kappa^4, \beta = \omega(2k^2 - l^2)/8\kappa^4, \gamma = -3\omega lk/4\kappa^4$ group velocity dispersion coefficients
- $\xi = \omega\kappa^2/2, \zeta = \omega(k^5 - k^3l^2 - 3kl^4 - 2k^4\kappa + 2k^2l^2\kappa + 2l^4\kappa)/2\kappa^2(k - 2\kappa)$

Numerical simulation of system (45),(46)

Notation

1. $A' = A/\kappa, B' = B/\kappa, t' = \omega t, x' = \kappa x, y' = \kappa y, \kappa = \sqrt{k^2 + l^2}$
2. $A = B = 0.1/\kappa + \epsilon, \epsilon = O(10^{-3}/\kappa)$ to give seed for any instability

Typical Data from Ocean Waves:

Typical wave frequency 0.09 Hz, $\omega = 0.56 s^{-1}, \kappa \approx 0.033 m^{-1}$. In next figures we view the results where $|A| = |B| \approx 3 = 0.1/\kappa$ Note in figure 4 at t=670sec A has localized wave packets with max amplitude approximately 10 meters.

4

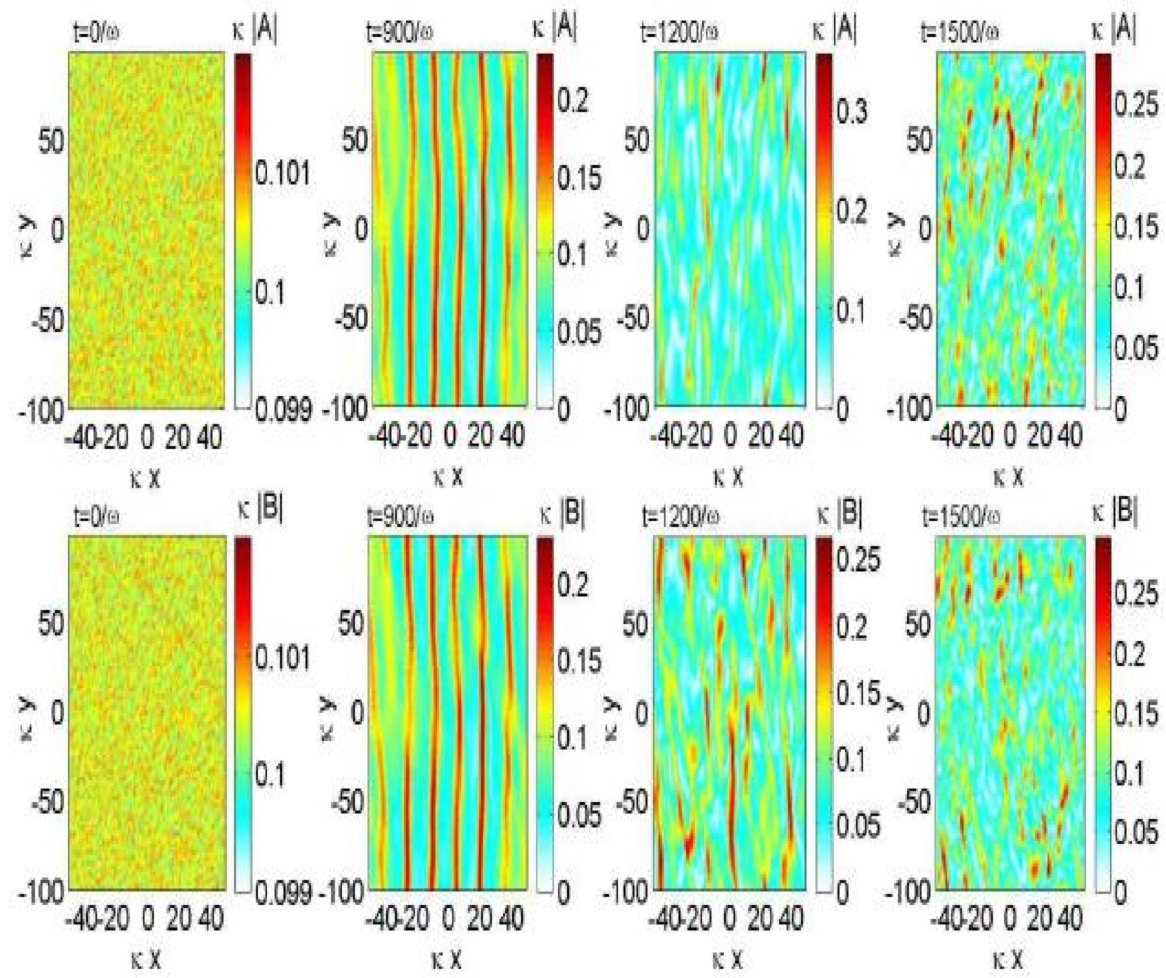


FIG. 4: The interaction between two waves, initially with equal amplitudes $|A| = |B| = 0.1\,\kappa^{-1}$ and a propagation angle of $\theta = \pi/8$ relative to the dichotome. Added to the initially homogeneous wave envelopes is a low-amplitude noise of order $10^{-3}/\kappa$ to give a seed to the modulational instability.

Quasi-Solitonic Turbulence [6]

Quasi Solitonic Turbulence corresponds to the time range from the beginning of instability till the formation of a freak wave event

MMT-model for gravity waves

$$i\frac{\partial \Psi}{\partial t} = |\frac{\partial}{\partial x}|^{1/2}\Psi + |\frac{\partial}{\partial x}|^{3/4}(||\frac{\partial}{\partial x}|^{3/4}\Psi|^2|\frac{\partial}{\partial x}|^{3/4}\Psi) \qquad (49)$$

Fourier transformed model

$$i\frac{\partial \hat{\psi}_k}{\partial t} = \omega_k \hat{\psi}_k + \int T_{123k}\hat{\psi}_1\hat{\psi}_2\hat{\psi}_3^*\delta(k_1+k_2-k_3-k)dk_1dk_2dk_3 \qquad (50)$$

where:

$\omega_k = |k|^\alpha$ Linear frequency parameter
$T_{123k} = \lambda |k_1 k_2 k_3|^{\beta/4}$ interaction parameter.
$\lambda = \pm 1$ balance between dispersive and non linear effects

Exact solution:

$$\Psi = Ae^{-kx-\omega t} \tag{51}$$

$$\omega = k^{1/2}(1 + k^{5/2}A^2) \tag{52}$$

- This solution can be constructed as a model of the Stokes wave.
- unstable with respect to modulational instability.

Numerical analysis of B-F instability development

1. the unstable monochromatic wave decomposes to a system of almost equal quasisolitons.

2. quasisolitonic turbulence is formed: quasisolitons move chaotically, interact with each other, and merge.

3. quasisolitons create one large quasisoliton, which exceeds threshold of instability and collapses, creating a freak wave

Quasi solitons and weak turbulence

Seek for solitons of the form:

$$\hat{\psi}_k(t) = e^{i(\Omega - kV)t}\hat{\phi}_k \quad (53)$$

Then Ω, V constant imply:

$$\hat{\phi}_k = -\frac{1}{\Omega - kV + \omega_k}\int T_{123k}\hat{\phi}_1\hat{\phi}_2\hat{\phi}_3^*\delta(k_1 + k_2 - k_3 - k)dk_1dk_2dk_3 \quad (54)$$

Quasi solitons are approximate solutions of (53) which look like envelope solitons

In the limit of a narrow spectrum centered at $k = k_m$, such as $\Omega - k_m V + k_m^\alpha \neq 0$ they are given by the formula:

$$\psi(x,t) \simeq \phi(x - Vt)e^{i\Omega t + ik_m(x - Vt)} \tag{55}$$

where

- $\phi(\xi) = \sqrt{\frac{\alpha(1-a)\kappa}{k_m^{\beta-\alpha+2}\cosh(\kappa\xi)}}$ for $\kappa = |k - k_m| << k_m$
- $\Omega = -(1-\alpha)k_m^\alpha - (1/2)\alpha(1-\alpha)k_m^{\alpha-2}\kappa^2$
- $V = \alpha k_m^{\alpha-1}$

1. If κ/k_m is small quasi solitons look almost like true solitons and can persist for a long time

2. If κ/k_m is large they can become unstable and develop into wave collapse.

FIGURE 5. $\beta = 3, \lambda = +1$. Snapshot of a quasisoliton at $x \simeq 3.7$ and $t = 10880$.

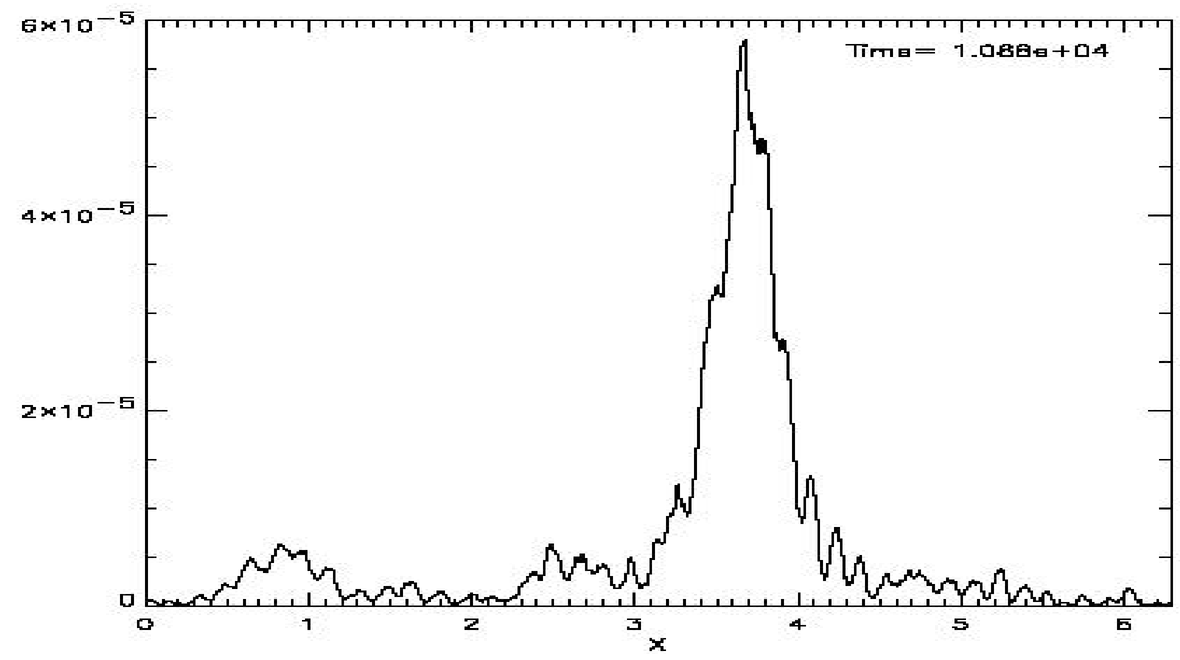
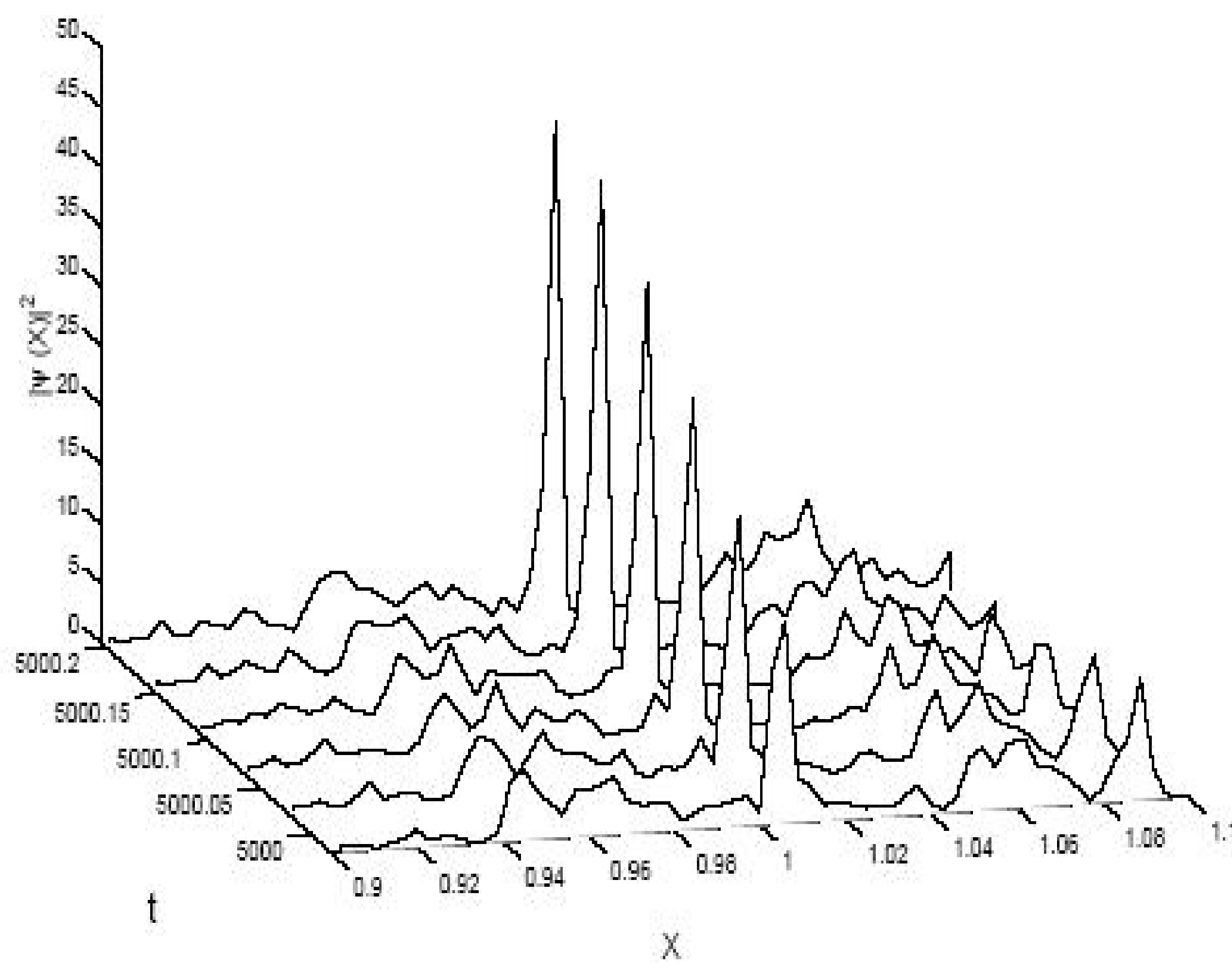


FIGURE 4. $\beta = 0, \lambda = -1$. Evolution towards collapse at $x \simeq 1$ between $t = 4999.980$ and $t = 5000.205$.



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