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Asymptotische Methoden der angewandten Mathematik — WS 2012-13

Exercise sheet 1

will be discussed on: Oct. 19, 2012

Problem 1: Show:

1. $\tan(x) = x + o(x^2)$ for $x \to 0$ without using the Taylor series for $\tan(x)$
2. $e^{o(x)} = 1 + o(x)$ for $x \to 0$ without any condition on the smoothess of $o(x)$ in x
3. $o(f(x)g(x)) = f(x)o(g(x))$ for $x \to x_0$

Problem 2: Prove or disprove:

$$(f = O(g), g = O(f) \text{ for } x \to x_0) \Rightarrow (\exists K \neq 0 : f \sim Kg \text{ for } x \to x_0).$$

Problem 3: Show that for $x > 1$ teh series $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} e^{-xt} t^{2n}$ converges uniformly (in t) on $\mathbb{R}$.

Problem 4: Order the following functions in an asymptotic sequence for $\varepsilon \to 0$:

$$\varepsilon^2, \sqrt{|\varepsilon|}, \log \left| \log \tfrac{1}{|\varepsilon|} \right|, 1, \sqrt{|\varepsilon|} \log \tfrac{1}{|\varepsilon|}, \varepsilon \log \tfrac{1}{|\varepsilon|}, \log \tfrac{1}{|\varepsilon|}, |\varepsilon|^{3/2}, \varepsilon^2 \log \tfrac{1}{|\varepsilon|}$$

Problem 5: Verify that

$$1 - \frac{e^{-x^2}}{\sqrt{\pi}} \left(\frac{1}{x} + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{2^n} \frac{1}{x^{2n+1}} \right)$$

with $(2n-1)!! := 1 \cdot 3 \cdot \dots \cdot (2n-1)$ is an asymptotic expansion for the error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

for $x \to \infty$.

Hints: Write first $\operatorname{erf}(x)$ as $\operatorname{erf}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$ and use the transformation $s = t^2$. Define then for $n \geq 0$

$$F_n(x) = \int_{x^2}^{\infty} s^{-n-1/2} e^{-s} ds,$$

derive a recursive relation between F_n and F_{n+1} and use it.