Interaction and Evolution of Two-Colour Nematicons in the Local Response Regime

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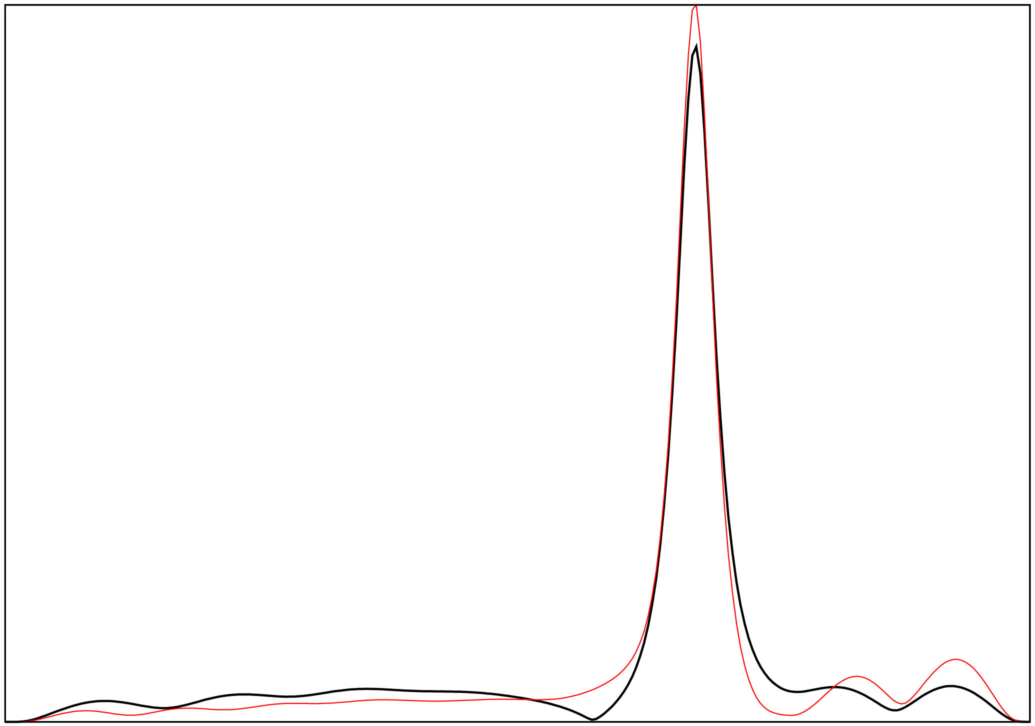
Two-Colour Nematicons



- Nematic Liquid Crystals

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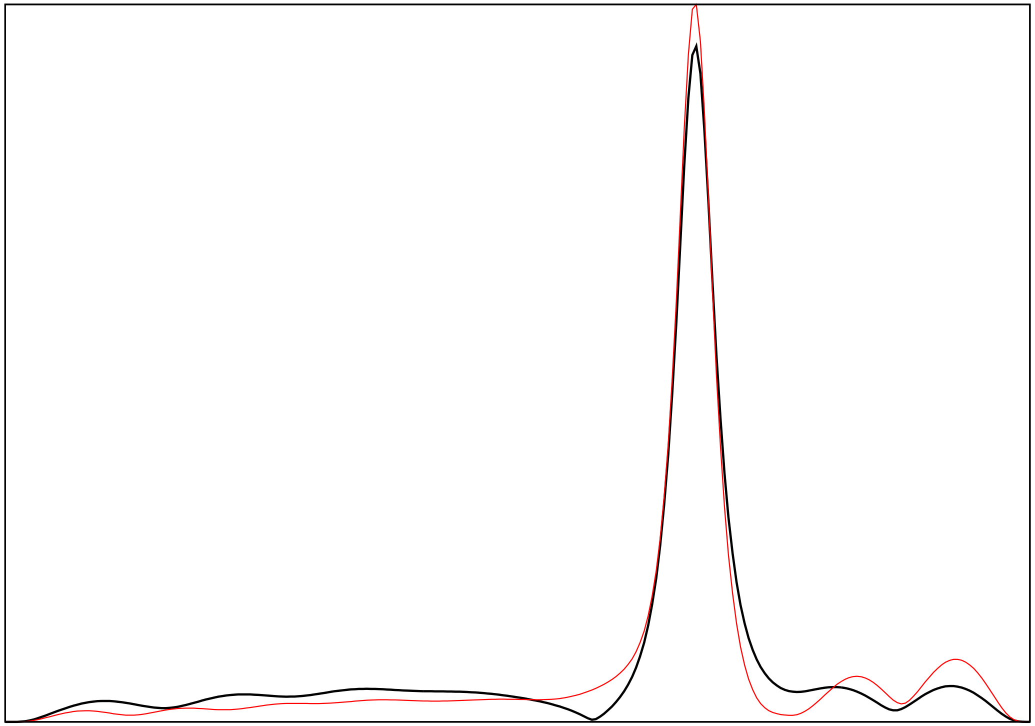

- Nematic Liquid Crystals
- Experimental Results

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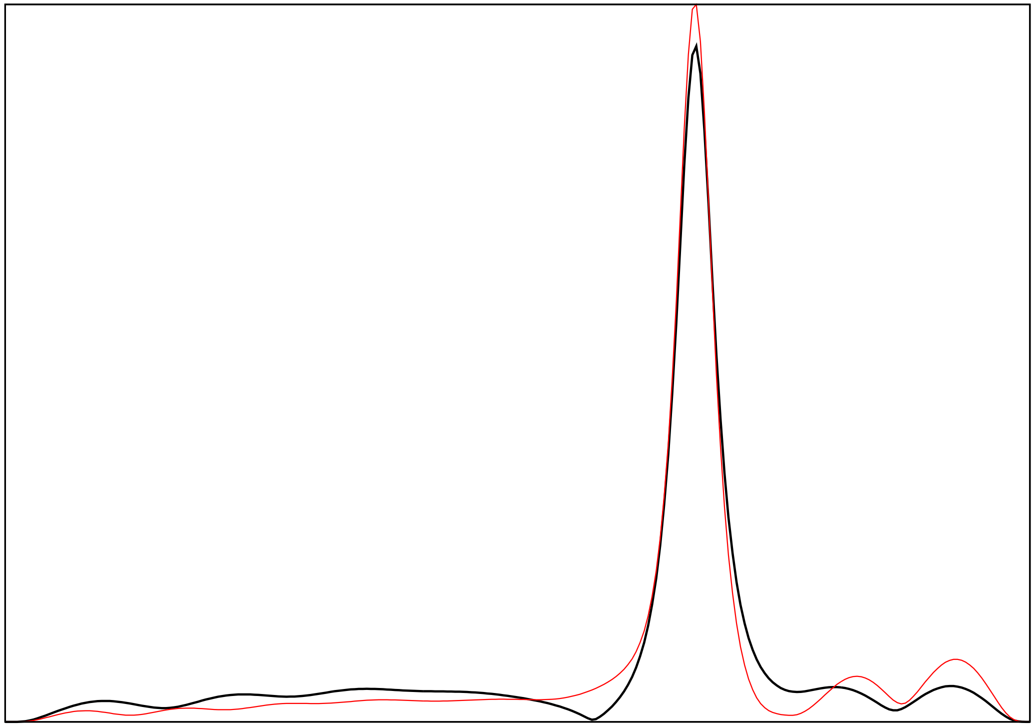

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- Governing Equations

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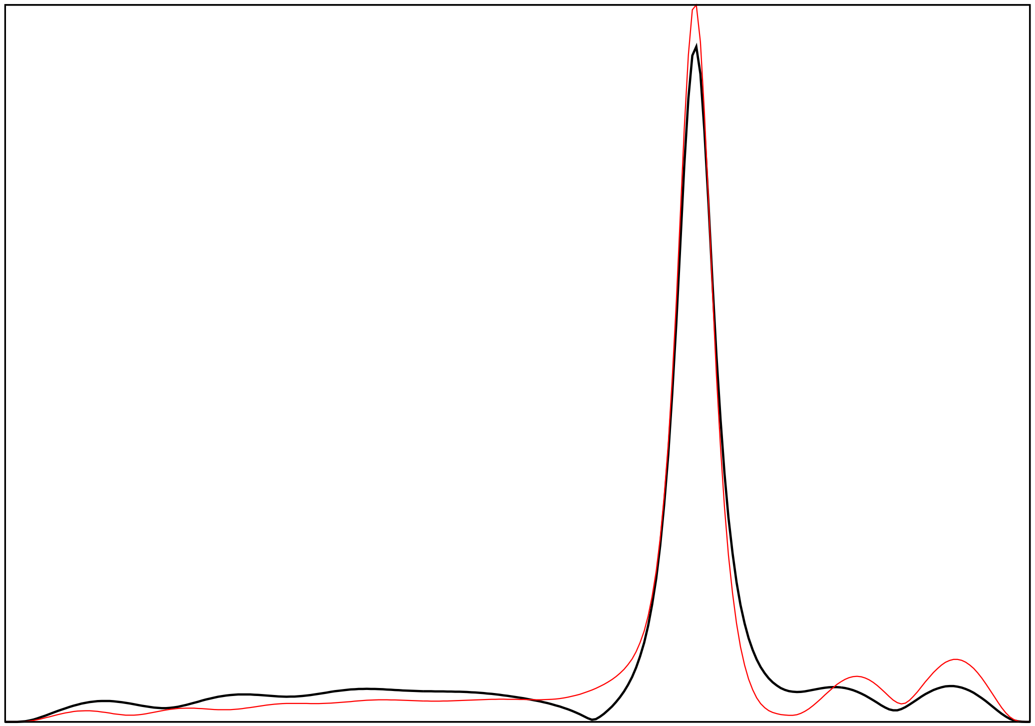

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- Governing Equations
- Variational Equations

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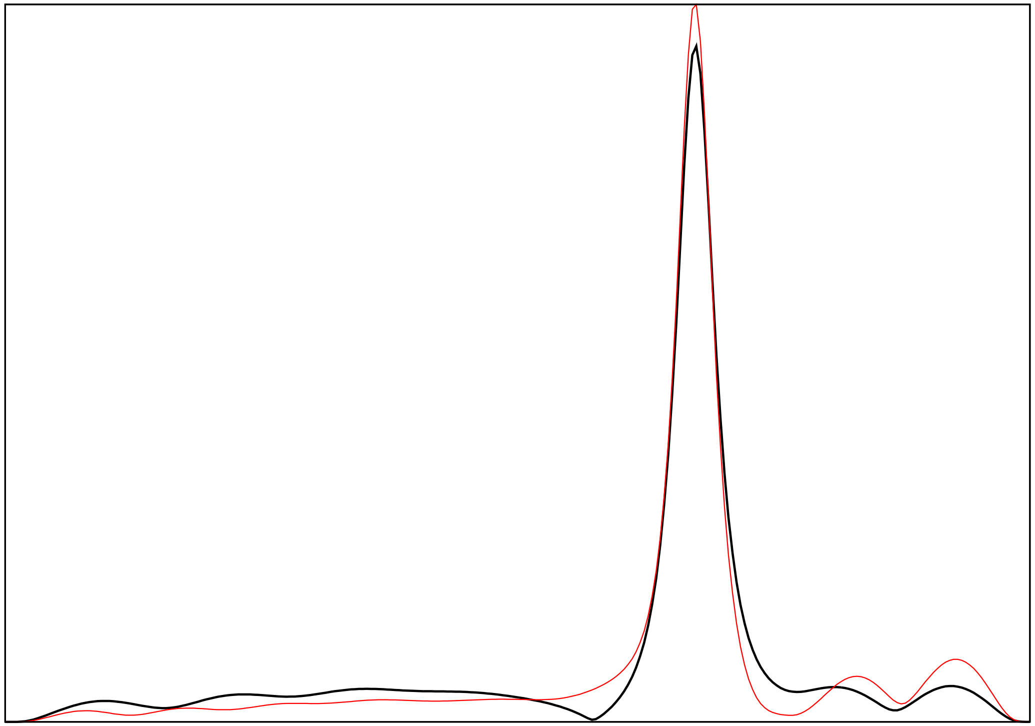

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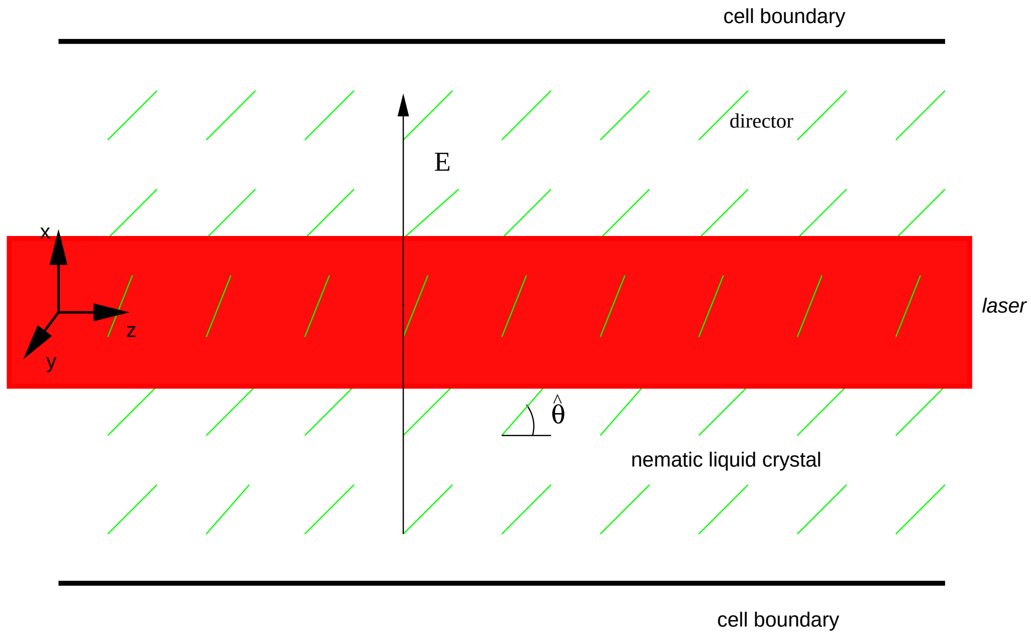
- Governing Equations
- Variational Equations
- Results

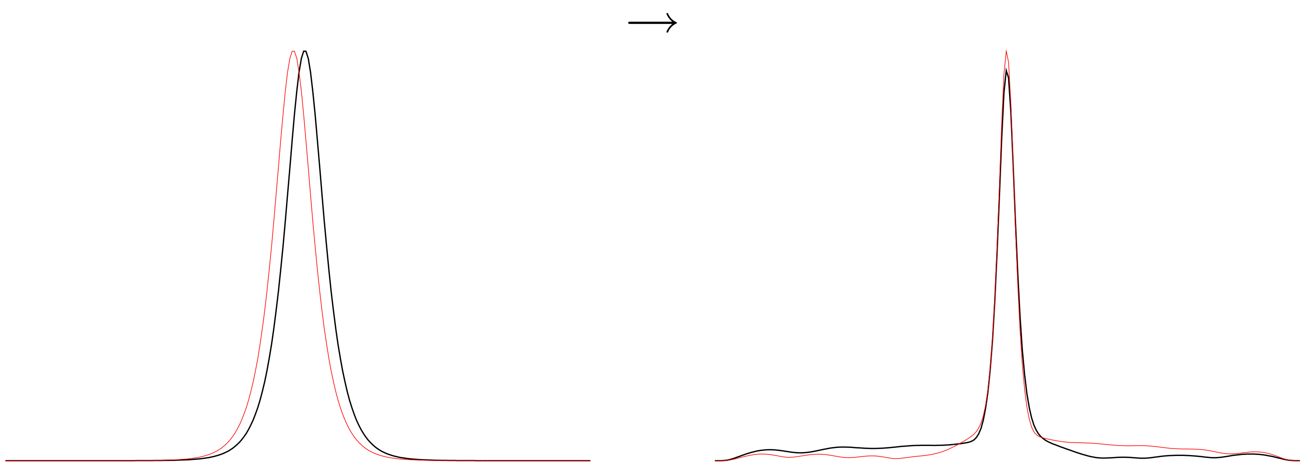
NLCs and Nematicons



- Nematic liquid crystal: rod-like molecules.
- Molecules tend to re-orientate in direction of electric field.
- Coherent, polarised optical beam.
- Beam intensity reduces angle of nematic molecules w.r.t. electric field.
- Beam self focuses.

Two-Colour Nematicons

- Different wavelengths ⇒ different degrees of birefringence and dispersion.
- Cross-phase modulation (CPM) negates this effect.
- Self-localised VS forms.



Governing Equations

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- Single Nematicon Governing Equations
  - NLS-like equation for electric field of the light.
  - Forced heat equation for the director.
- Two-Colour Nematicon Governing Equations
  - CNLS-like equation for each colour

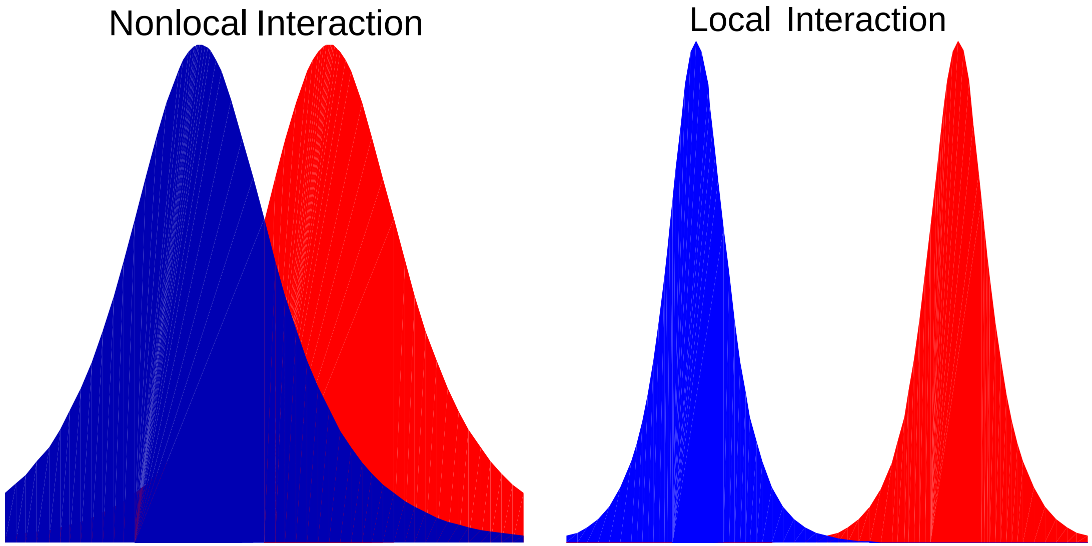
$$
\begin{aligned}
iu_t + \frac{D_1}{2}\nabla^2 u + Au\sin 2\theta &= 0\\
iv_t + \frac{D_2}{2}\nabla^2 v + Bv\sin 2\theta &= 0.
\end{aligned}
$$

  - Director equation

$$
q\sin 2\theta - 2A|u|^2\cos 2\theta - 2B|v|^2\cos 2\theta = \nu\nabla^2\theta
$$

ν large and ν small

- ν: ratio of elastic energy of nematicon to energy of electric field. Nonlocality parameter.



ν large and ν small

- ν: ratio of elastic energy of nematicon to energy of electric field. Nonlocality parameter.
- $\nu \rightarrow \infty$ for nonlocal crystals.
  - Low power input beam. Studied experimentally extensively.
  - Nonlocality stops collapse.
- $\nu \rightarrow 0$ for local crystals.
  - Strongly illuminated or heated NLCs.
  - Stability caused by saturation of NLC director motion.

ν small

Consider director equation for ν small <-

Rearrange

$$\tan 2\theta = \frac{2\left(A|u|^2 + B|v|^2\right)}{q}$$

Assuming radial symmetry, the governing equations are

$$iu_t + \frac{D_1}{2}u_{rr} + \frac{D_1}{2r}u_r + \boxed{Au\left(\frac{2\left(A|u|^2+B|v|^2\right)}{\sqrt{q^2+4(A|u|^2+B|v|^2)}}\right)} = 0$$

$$iv_t + \frac{D_2}{2}v_{rr} + \frac{D_2}{2r}v_r + \boxed{Bv\left(\frac{2\left(A|u|^2+B|v|^2\right)}{\sqrt{q^2+4(A|u|^2+B|v|^2)}}\right)} = 0.$$

Saturable nonlinearity

Modulation Equations

The Lagrangian for these equations is

$$
\begin{aligned}
L &= ir\left(\bar{u}u_t - u\bar{u}_t\right) + ir\left(\bar{v}v_t - v\bar{v}_t\right) - D_1 r|u_r|^2 - \\
&\quad D_2 r|v_r|^2 + r\sqrt{q^2 + 4\left(A|u|^2 + B|v|^2\right)} - rq.
\end{aligned}
$$

Next steps:

- Insert trial functions into Lagrangian.

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Next steps:

- Insert trial functions into Lagrangian.
- Average Lagrangian $\int_{-\infty}^{\infty} L = 0$.
- Variational equation $\longrightarrow$ Modulation Equations.
- Plus various conservation equations.

Nematicon Equations

Trial Functions

$$u = a_1 \operatorname{sech}\left(\frac{\sqrt{(x-\xi_1)^2 + y^2}}{w_1}\right) e^{i\sigma_1 + iV_1(x-\xi_1)} + ig_1 e^{i\sigma_1 + iV_1(x-\xi_1)}$$

$$v = a_2 \operatorname{sech}\left(\frac{\sqrt{(x-\xi_2)^2 + y^2}}{w_2}\right) e^{i\sigma_2 + iV_2(x-\xi_2)} + ig_2 e^{i\sigma_2 + iV_2(x-\xi_2)}.$$

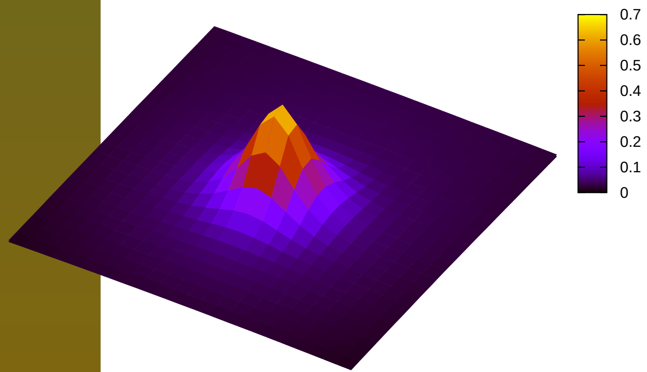
Alternative trial functions

$$u = a_1 e^{-\frac{(x-\xi_1)^2 + y^2}{w_1^2}} e^{i\sigma_1 + iV_1(x-\xi_1)} + ig_1 e^{i\sigma_1 + iV_1(x-\xi_1)}$$

$$v = a_2 e^{-\frac{(x-\xi_2)^2 + y^2}{w_2^2}} e^{i\sigma_2 + iV_2(x-\xi_2)} + ig_2 e^{i\sigma_2 + iV_2(x-\xi_2)}.$$

Nematicon Equations

- The first terms represent varying soliton-like pulses.
- Second terms represent low wavenumber, low phase-speed radiation shed by the pulses. Should form circular shelves under the pulses.



where

- a_k: amplitude
- w_k: width
- ξ_k: displacement from x
- σ_k: phase
- V_k:
- g_k: shelf amplitude

Equivalent Gaussians

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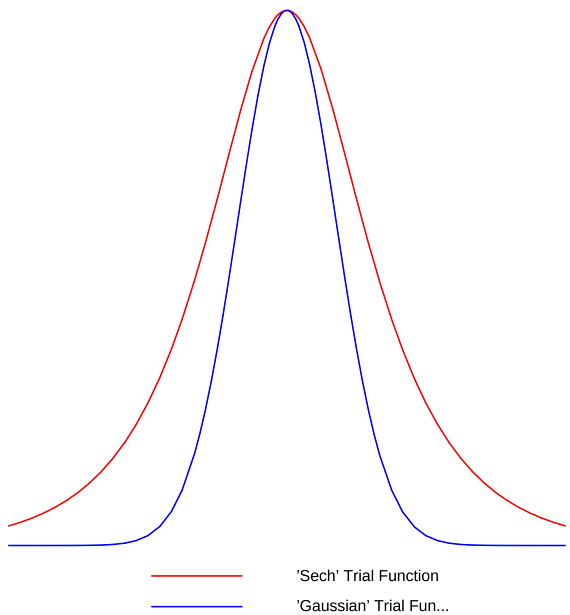

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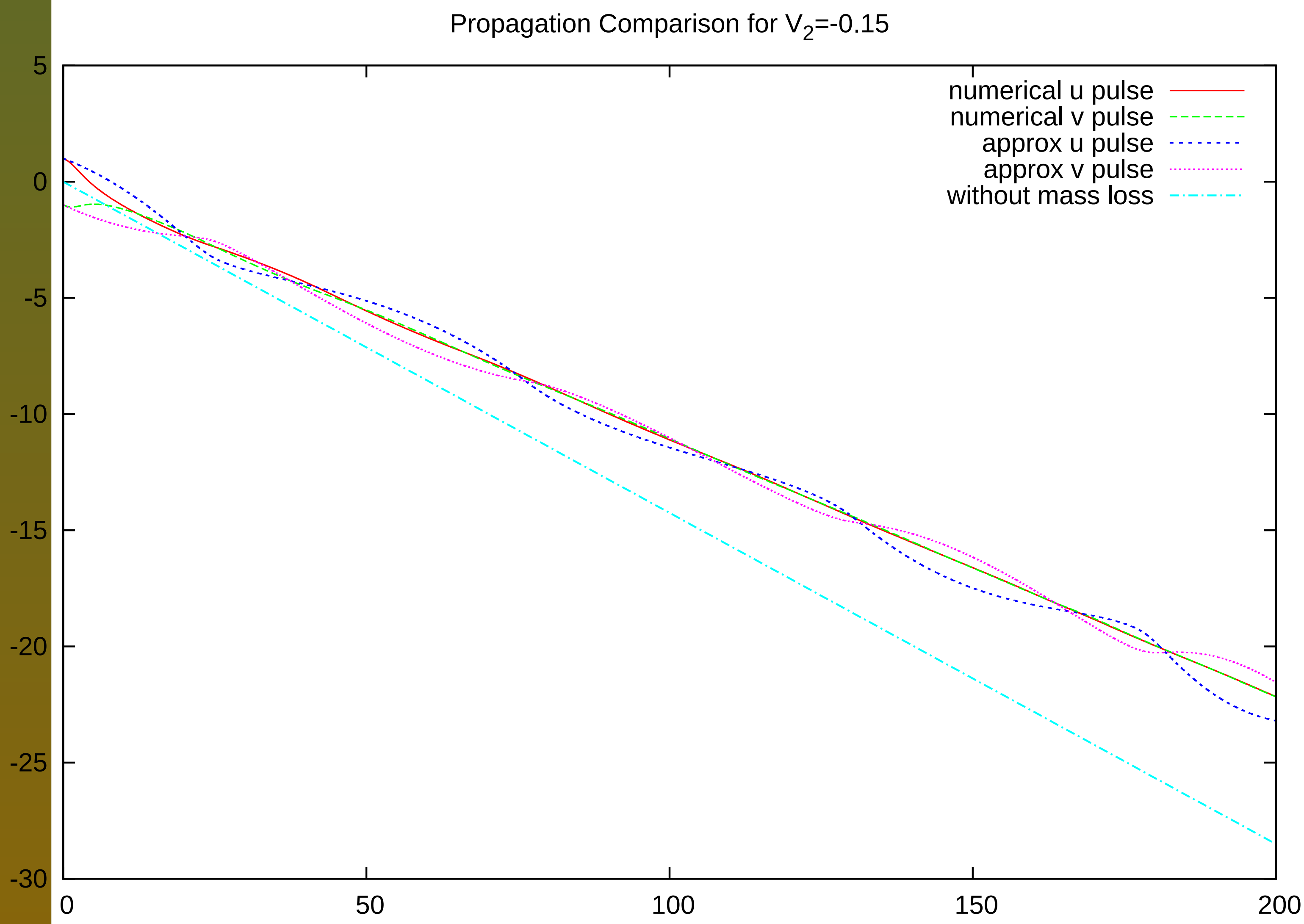
Radiation Loss

- Shed radiation has small amplitude relative to the nematicons.
- Governed by linearised NLS equations

$$
\begin{aligned}
iu_z + \frac{D_1}{2r}\frac{\partial}{\partial r}(ru_r) &= 0 \\
iv_z + \frac{D_2}{2r}\frac{\partial}{\partial r}(rv_r) &= 0
\end{aligned}
$$

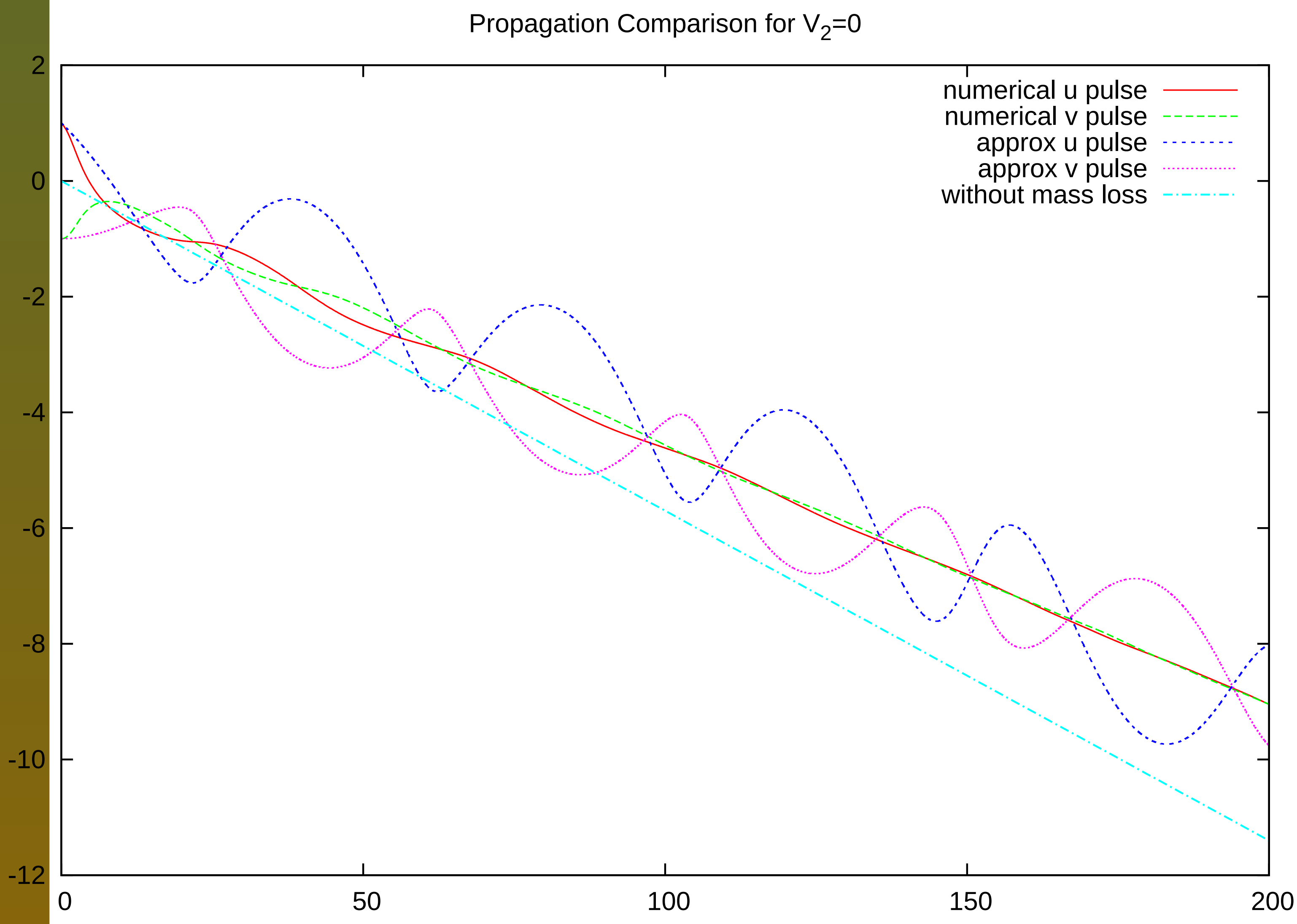
- Match shed radiation to existing radiation value at edge of shelf.
- Mass lost to the shed radiation important.
- Mass loss and radiation shelf are both initially 0.
- Add mass loss term to equation for g_i.

Results

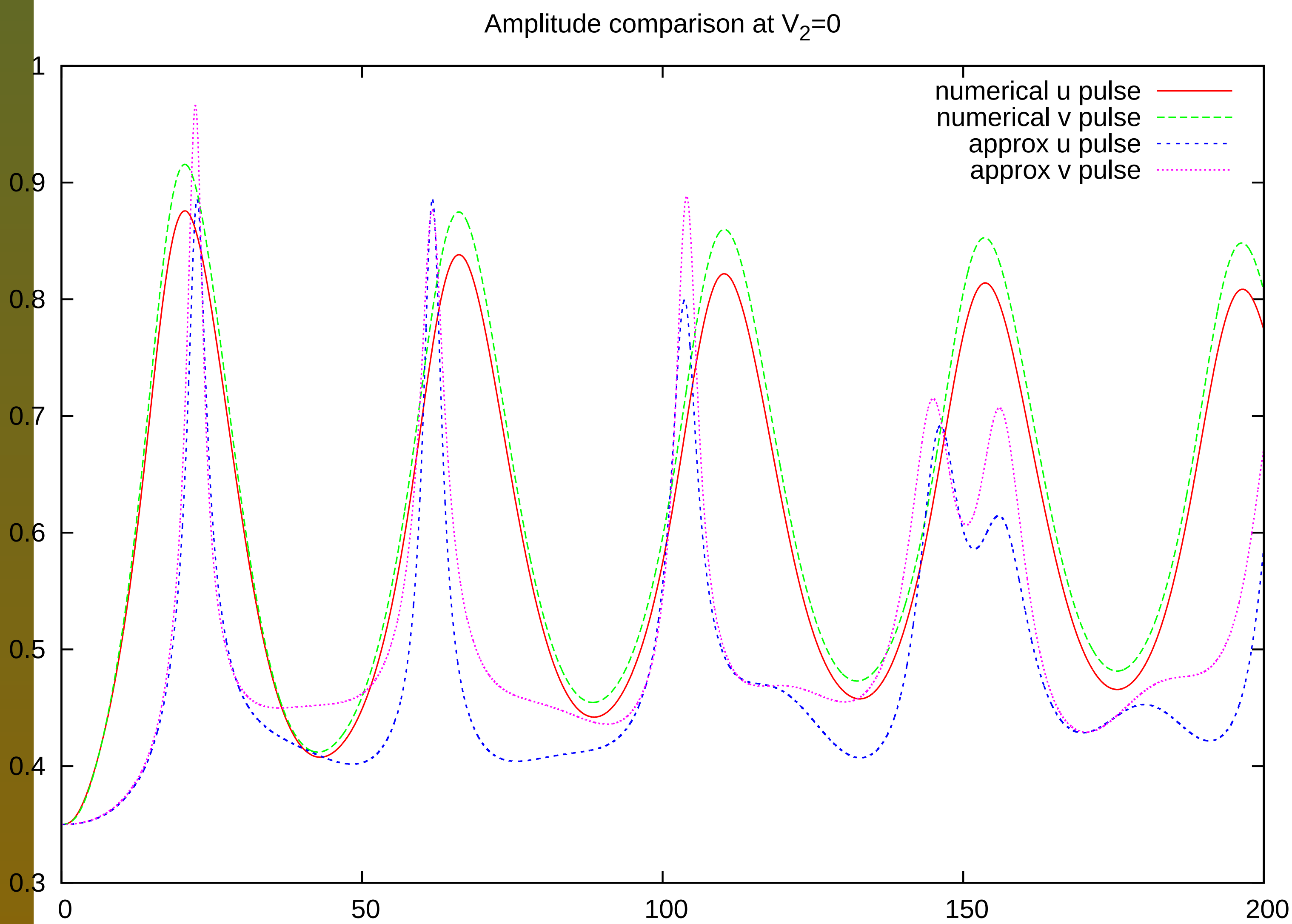


Propagation Comparison for V2=-0.15
numerical u pulse
numerical v pulse
approx u pulse
approx v pulse
without mass loss
5
0
-5
-10
-15
-20
-25
-30
0
50
100
150
200

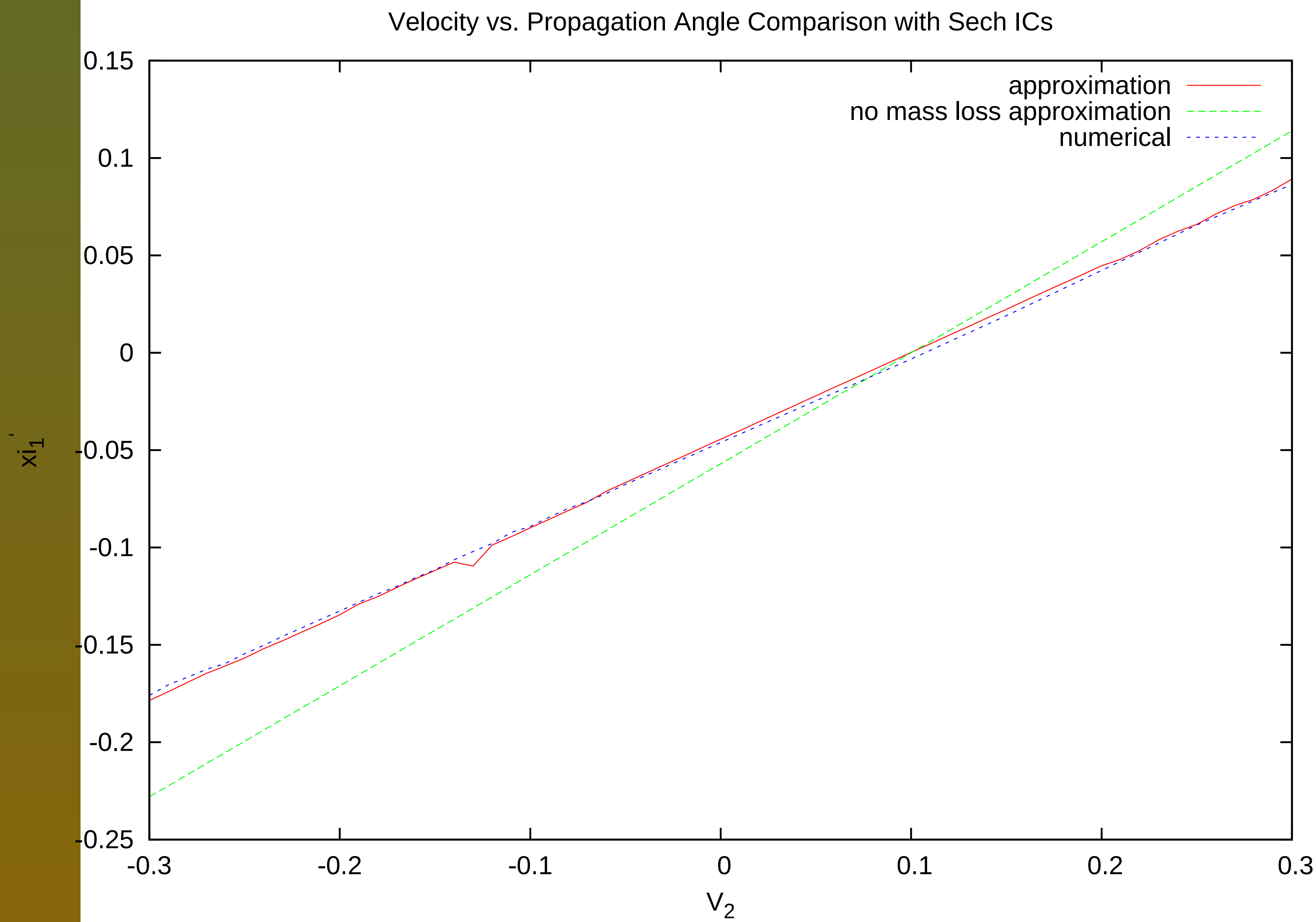
Results


Propagation Comparison for V2=0
numerical u pulse
numerical v pulse
approx u pulse
approx v pulse
without mass loss
2
0
-2
-4
-6
-8
-10
-12
0
50
100
150
200

Results

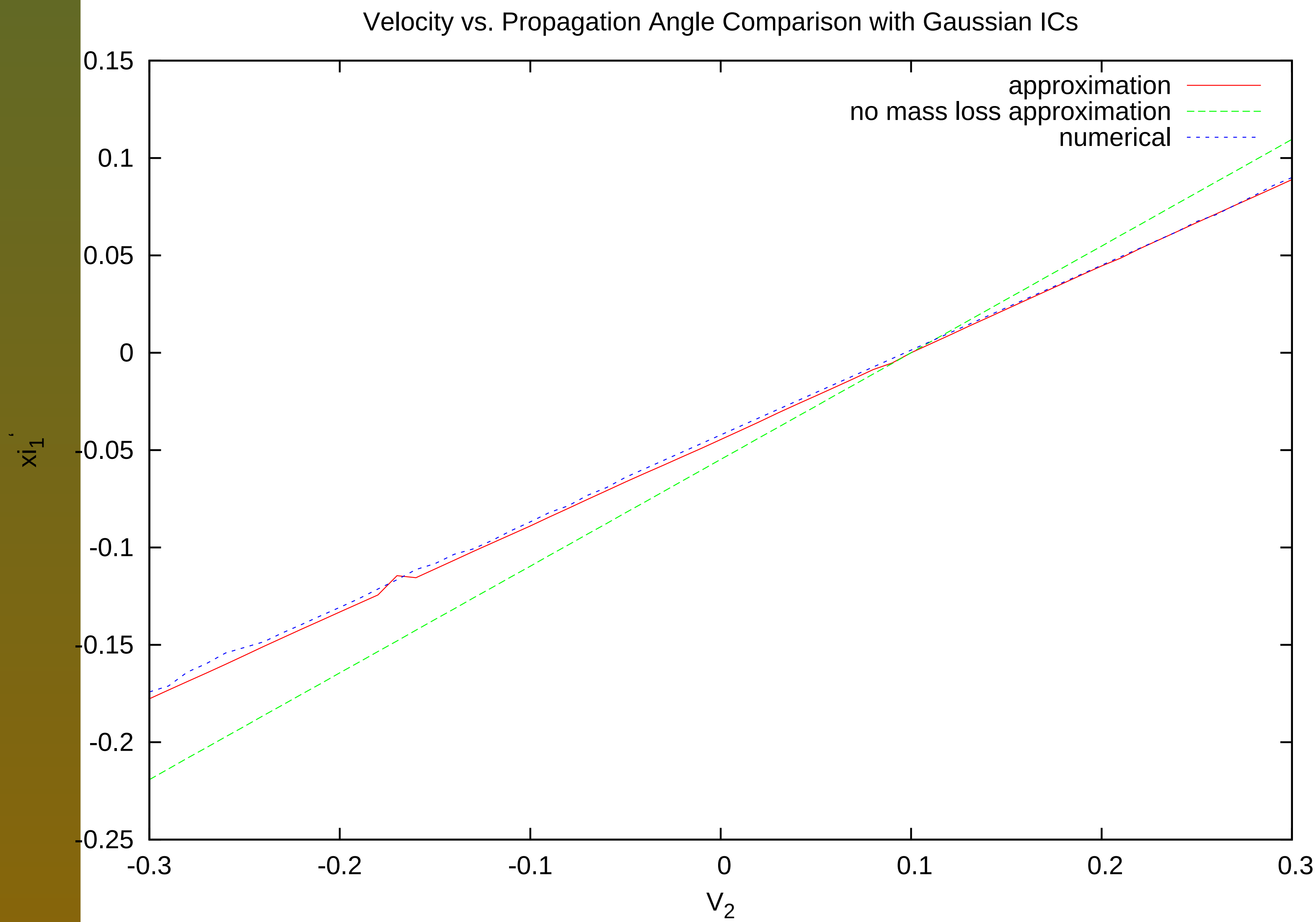

Amplitude comparison at V_2=0
numerical u pulse
numerical v pulse
approx u pulse
approx v pulse
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
50
100
150
200

Results

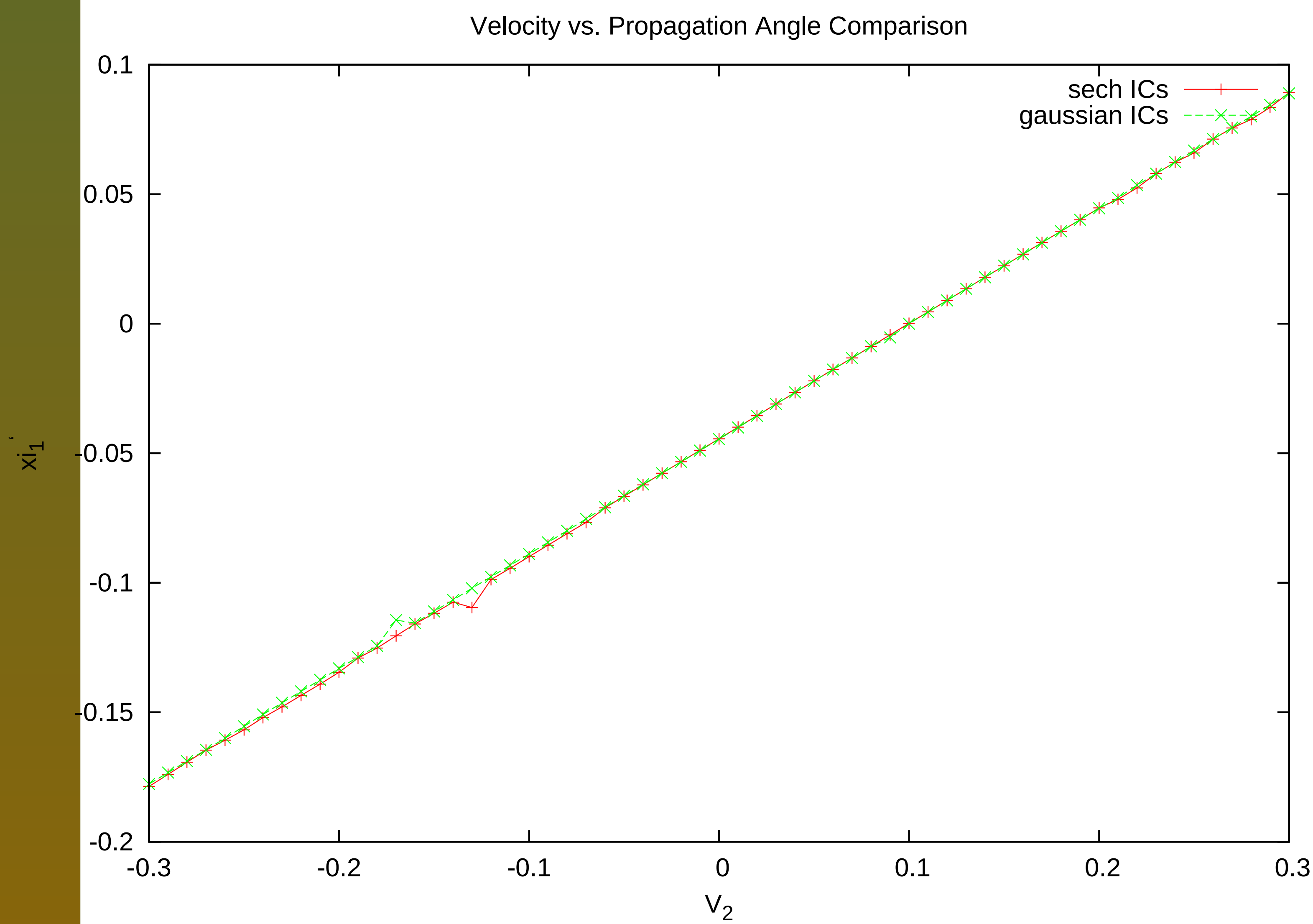


Velocity vs. Propagation Angle Comparison with Sech ICs
approximation
no mass loss approximation
numerical
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
xi1'
V2

Results

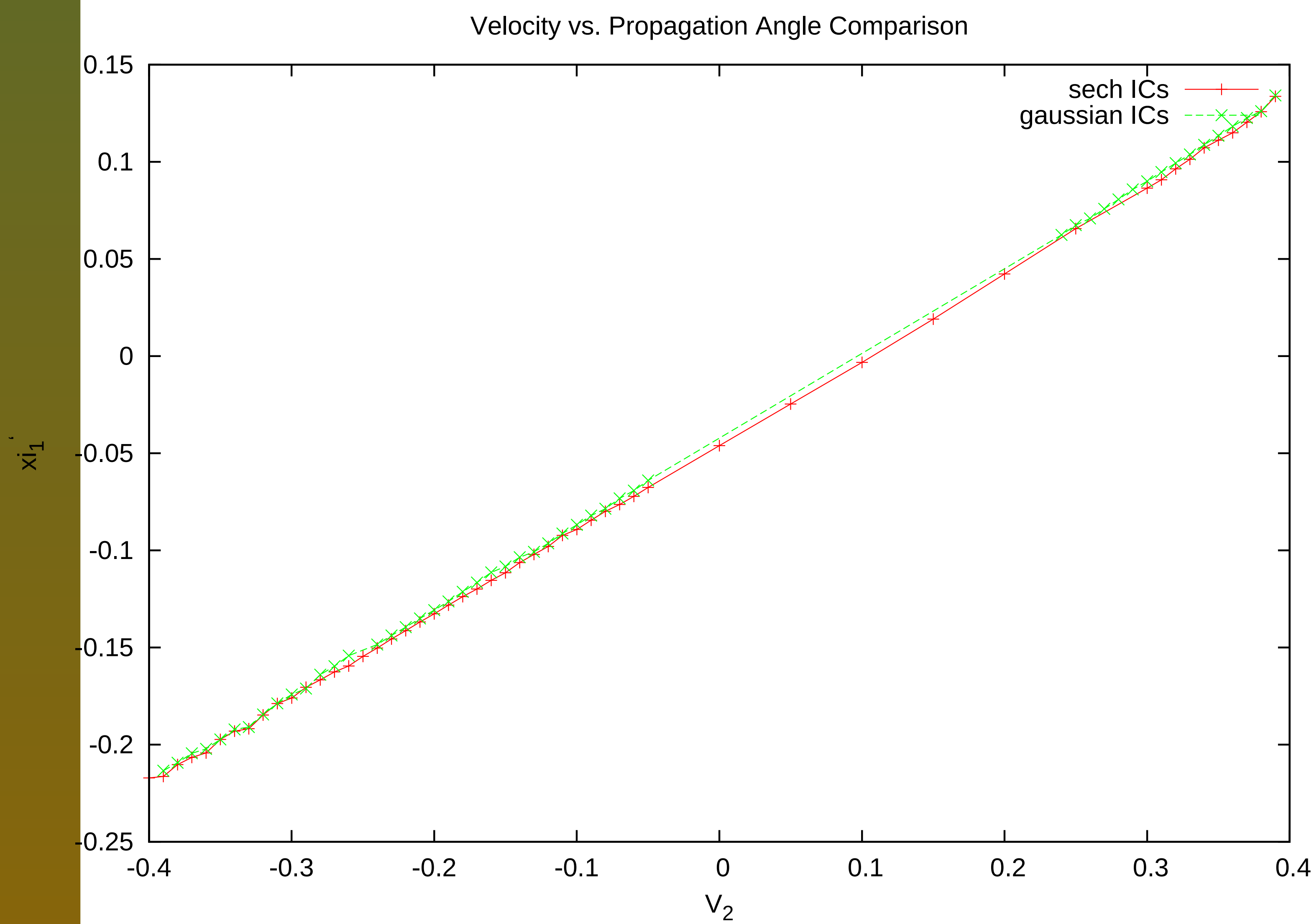

Velocity vs. Propagation Angle Comparison with Gaussian ICs
approximation
no mass loss approximation
numerical
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
xi₁'
V₂

Results



Velocity vs. Propagation Angle Comparison
sech ICs
gaussian ICs
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
xi'1
V2

Results



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sech ICs
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0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
xi₁'
V₂

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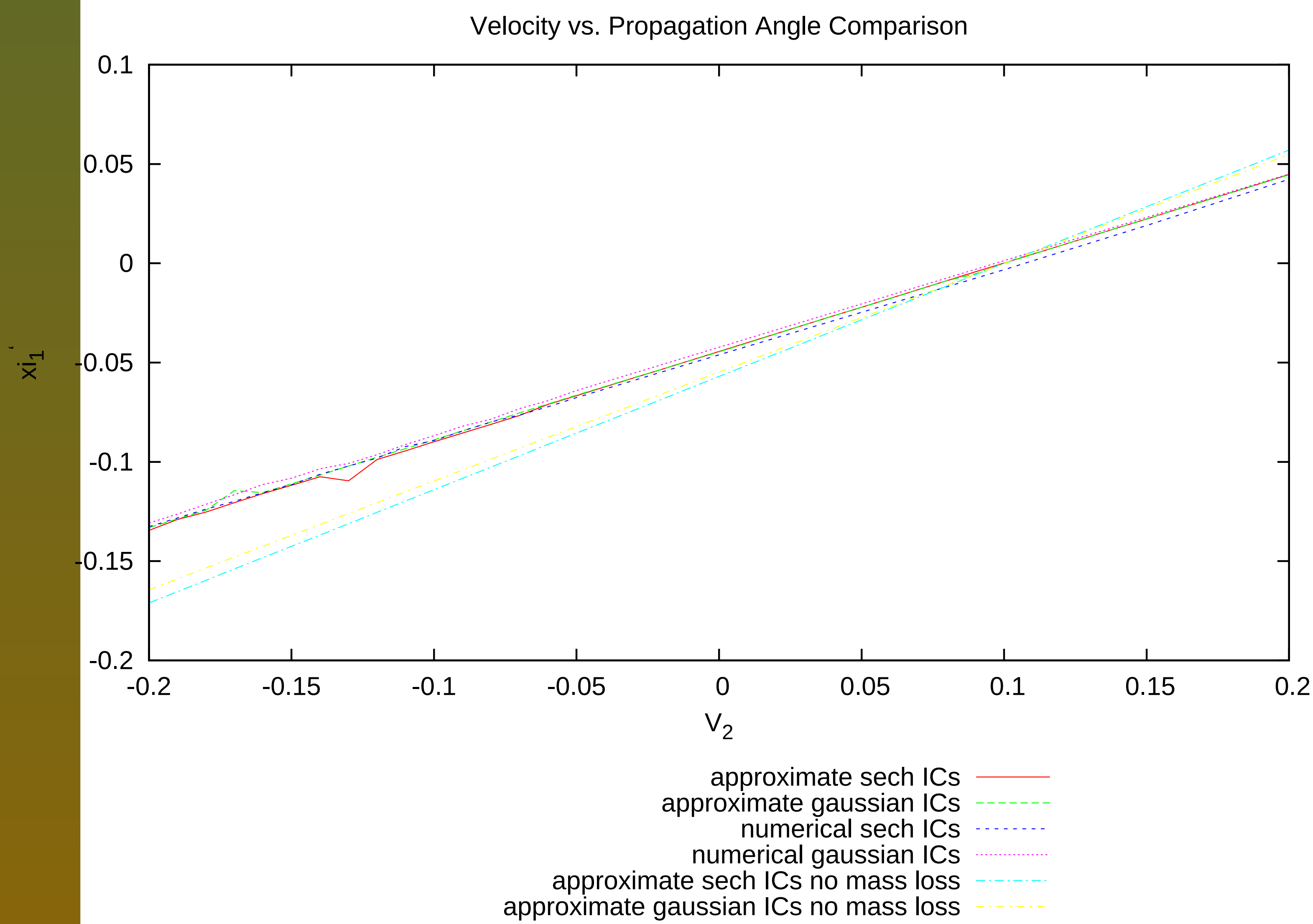

Velocity vs. Propagation Angle Comparison
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
xi$_1$'
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
V$_2$
approximate sech ICs
approximate gaussian ICs
numerical sech ICs
numerical gaussian ICs
approximate sech ICs no mass loss
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