Nonlinear Filamentation Dynamics

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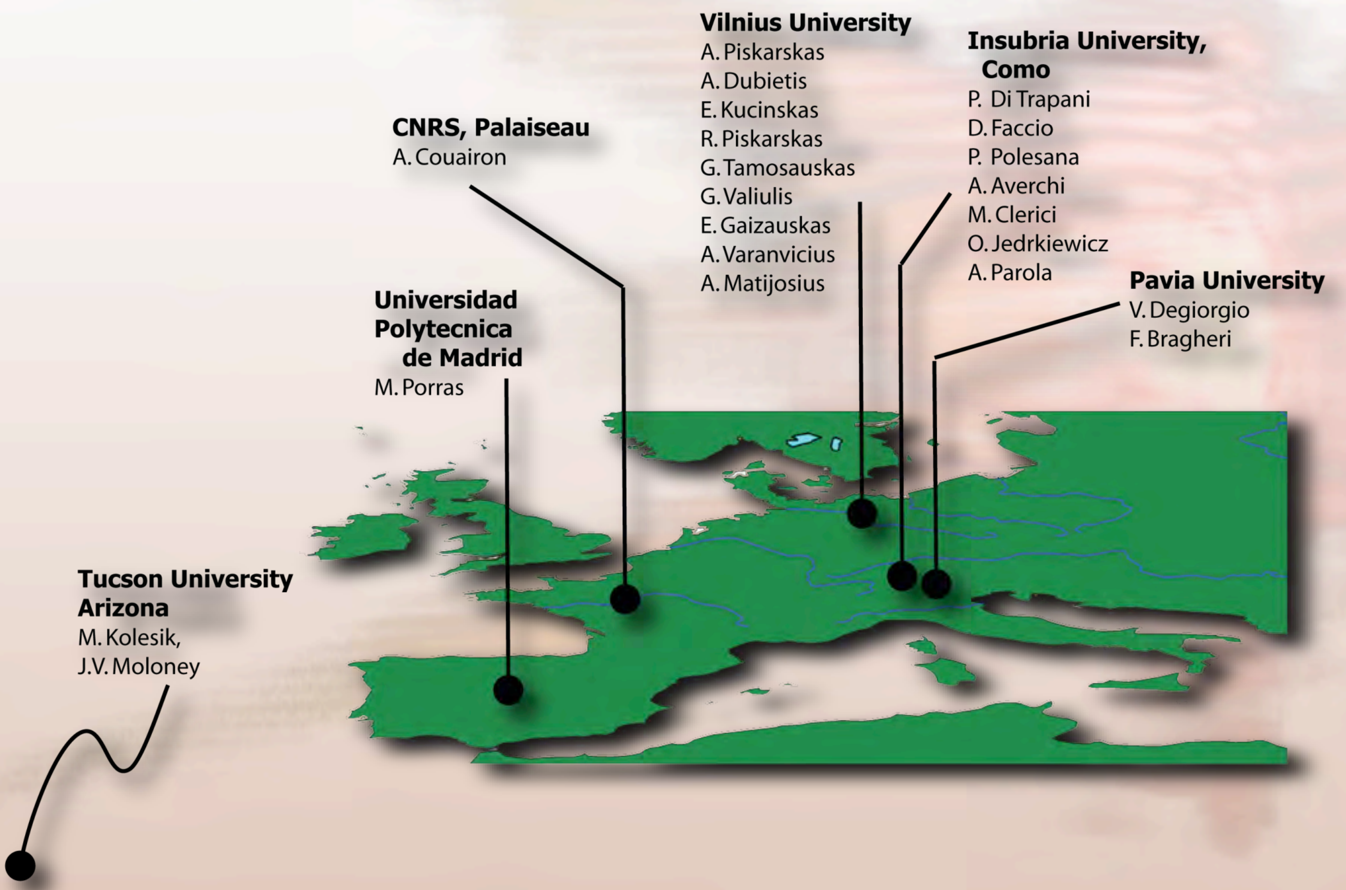


Ultrashort Laser Pulse Filamentation

Kerr nonlinearity gives an intensity dependent phase variation

$$n = n(r,t) = n_0 + n_2 I(r,t)$$

$$\phi = \phi(r,t) = k_0 n(r,t) z - \omega t$$

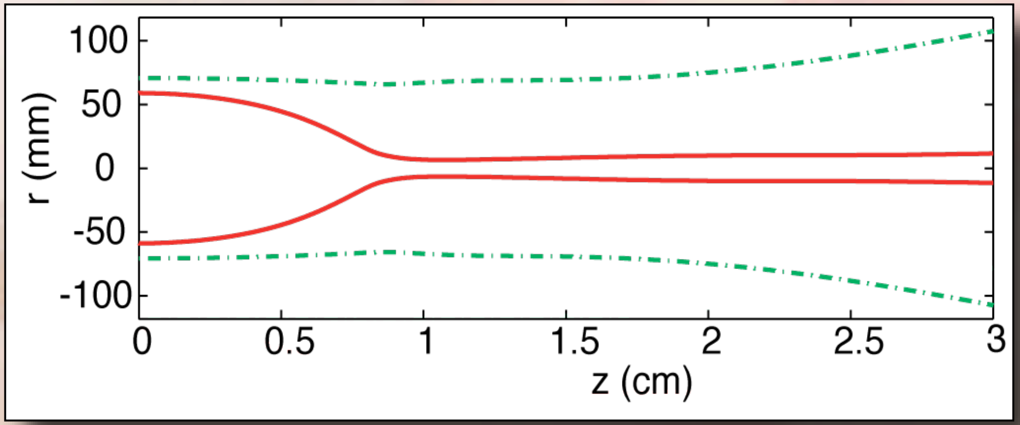
$$\Delta\omega = -\frac{d\phi(t)}{dt}$$

$$\Delta k = \frac{d\phi(r)}{dr}$$

If the laser pulse power is $P > P_{cr} = 3.77 \frac{\lambda^2}{8\pi n_0 n_2}$ the pulse will go toward catastrophic collapse

In real settings the collapse is arrested by various (competing) effects: NLL, plasma defocusing, GVD higher order nonlinearities etc...

The filament is characterized by the formation of high intensity peak whose fluence profile propagates without diffraction

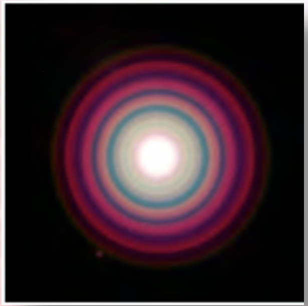


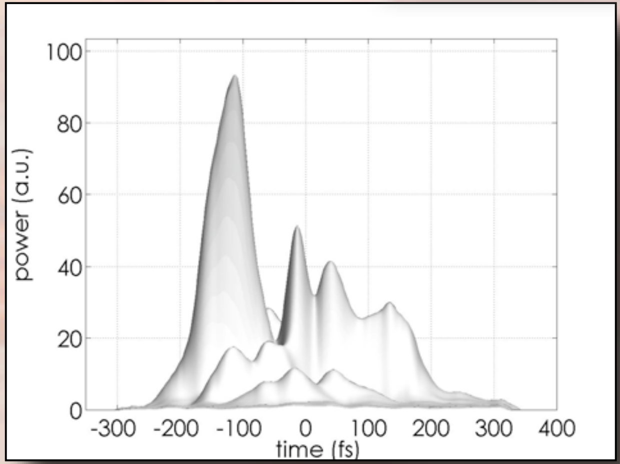
Ultrashort Laser Pulse Filamentation

Filaments show some universal features in all Kerr media...

Spectral features:
- axial supercontinuum (white light)
- conical emission

r,t features:
- localized, non-diffracting peak surrounded by large background
- pulse splitting
- shock front formation





Applications
- lightning protection
- nonlinear filamentation optics
  e.g. frequency conversion, pulse compression, HHG
-atmospheric LIDAR
- physics - connections with other systems
  e.g. BEC

Self guiding model,
Moving focus model,
Dynamical spatial replenishment,
Effective Three Wave Mixing Model....

X Wave Model

Filamentation is interpreted as a spontaneous generation and dynamical interaction of nonlinear X waves or conical waves:
X Waves are taken as the natural attractors for the pulse evolution (stationary states) and all physical interactions are treated as interactions between X (Conical) Waves

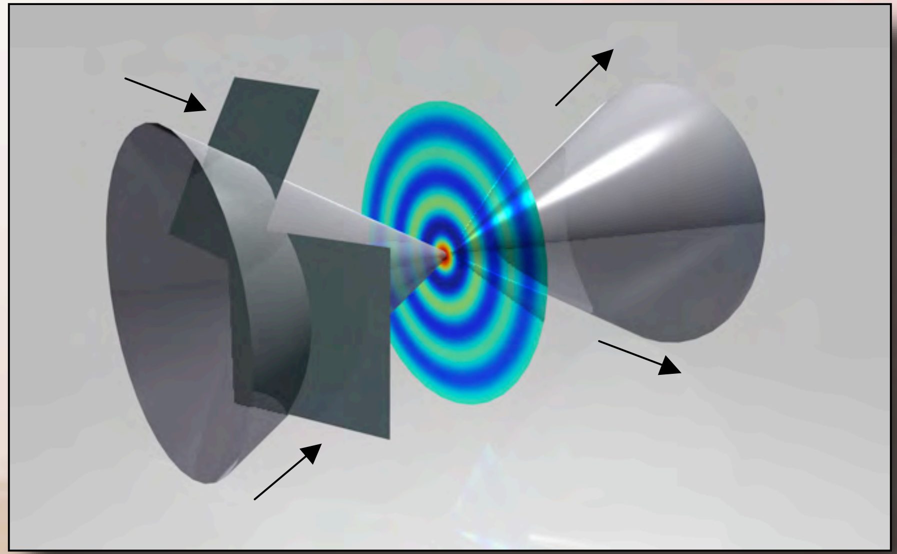
CONICAL WAVES ??? ...

Conical Waves

simplest conical wave → monochromatic → Bessel Beam

plane waves flowing
along a conical surface:
interference pattern = Bessel Beam

The central peak is ***non-diffracting***



Conical Waves

we write the polychromatic conical wave as a superposition of Bessel beams

$$\begin{aligned} A(r,z,t) &= \int d\omega S(\omega) J_0[k_\perp(\omega) r] e^{i\omega t} \\ k_\perp &= k \sin\theta = \sqrt{k^2 - k_z^2} \end{aligned}$$

By controlling angular dispersion it is possible to balance material GVD
→ NON DISPERSIVE PULSES

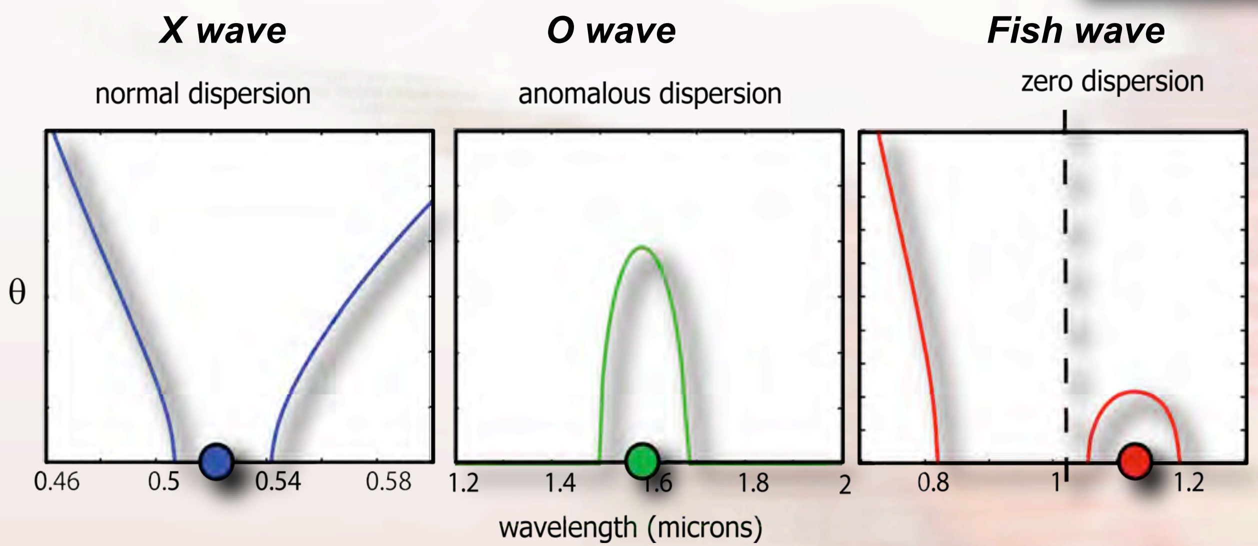
The requirement of non-dispersive propagation can be explicitly imposed by taking $k_z \propto \omega$
(so that $v_g = 1/dk_z/d\omega$ = const. and GVD = 0)

$$\begin{aligned} k_\perp &= \sqrt{k^2 - k_z^2} \\ k_z &= (k_0 - \beta) + (k_0' - \alpha)\Omega \end{aligned}$$

So it appears extemely convenient to describe conical pulses not in direct (r,t) space but rather in (k,ω) space.

Conical Waves

$$k_\perp = \sqrt{k^2 - k_z^2}$$
$$k_z = (k_0 - \beta) + (k_0' - \alpha)\Omega$$



2 important features:

- ***STATIONARITY*** in both linear and nonlinear regimes
- (independently) ***tunable phase and group velocities***

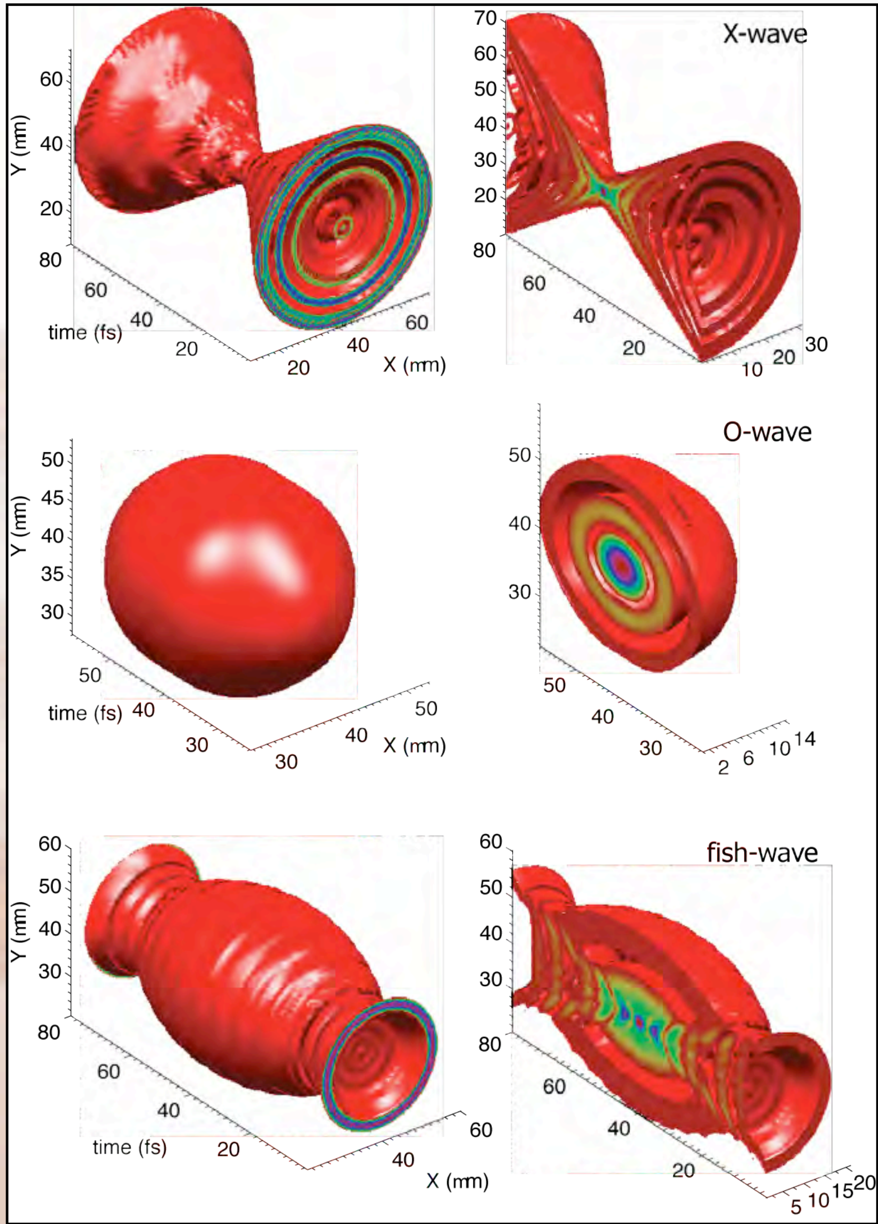
may be SUPER or SUB luminal

$$v_\phi = \frac{\omega}{k_0 - \beta}$$
$$v_g = \frac{1}{k_0' - \alpha}$$

Conical Waves

(r,t) profiles of “envelope” conical waves

$$
\begin{aligned}
A(r,z,t) &= \int_{-\infty}^{\infty} S(\Omega) J_0[k_\perp(\Omega) r] e^{j\Omega t} d\omega, \\
k_\perp &= \sqrt{k^2 - k_z^2} \\
k_z(\omega) &= [k(\omega_0) - \beta] + (k_0' - \alpha)\Omega
\end{aligned}
$$

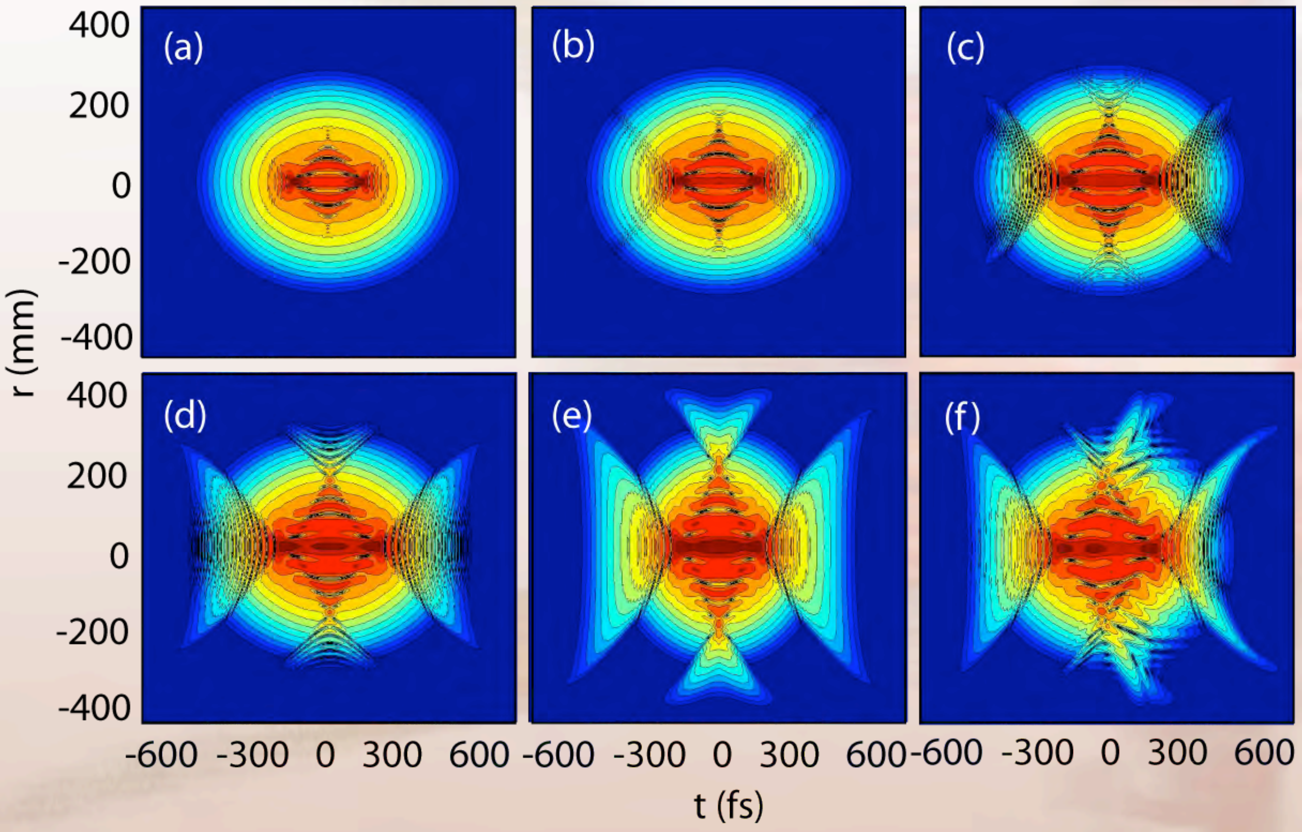


X waves, O waves, Fish waves

Optical Filaments & X Waves

near-field (r,t) distribution may be analyzed both experimentally (3D-mapping) and numerically
BUT...is this really the best possible quantity to characterize filaments?

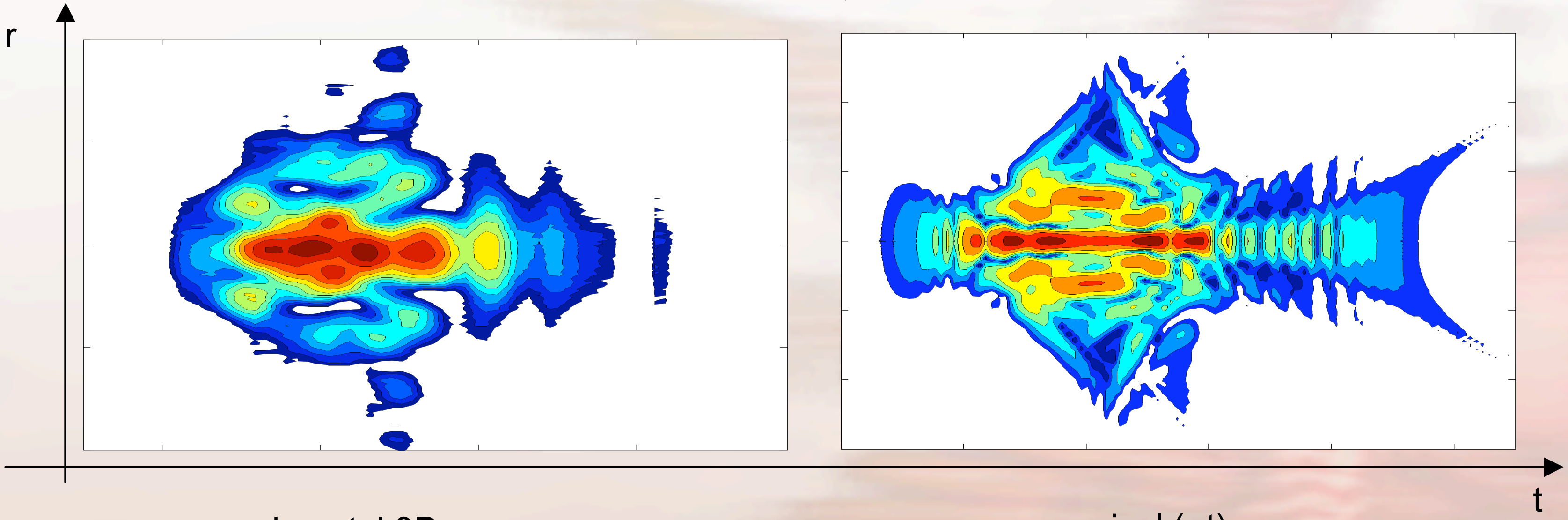
The initial stages of the filamentation process show a relatively clear and easy to interpret (r,t) profile....



Optical Filaments & X Waves

Further propagation leads to very complicated and difficult-to-interpret (r,t) profiles
.......practically useless!!!!

800 nm filament, 3 cm water



experimental 3D-map

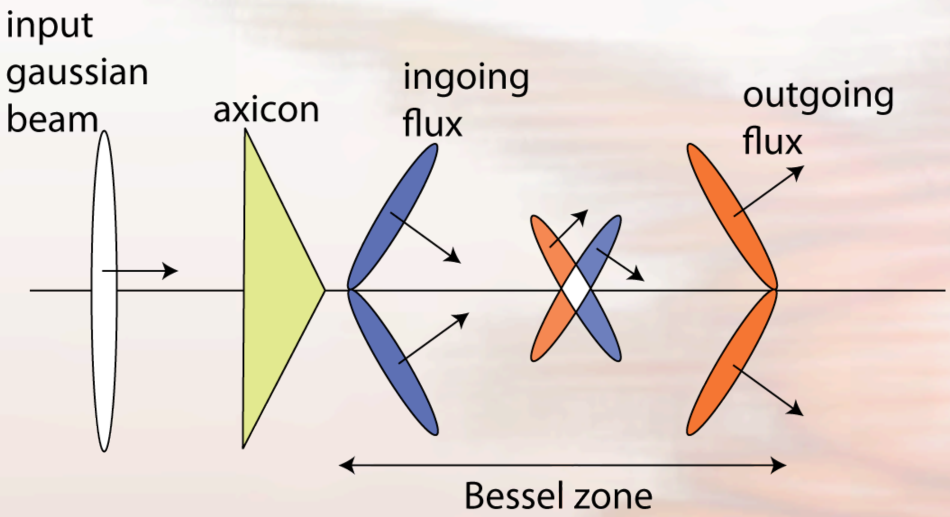
numerical (r,t)

So we need to find some other quantity that gives a clearer vision of the underlying physics
...search for the proof that X Waves are spontaneously formed during filamentation....

Optical Filaments & X Waves

instead of the intensity distribution, lets look at the energy flux distribution....

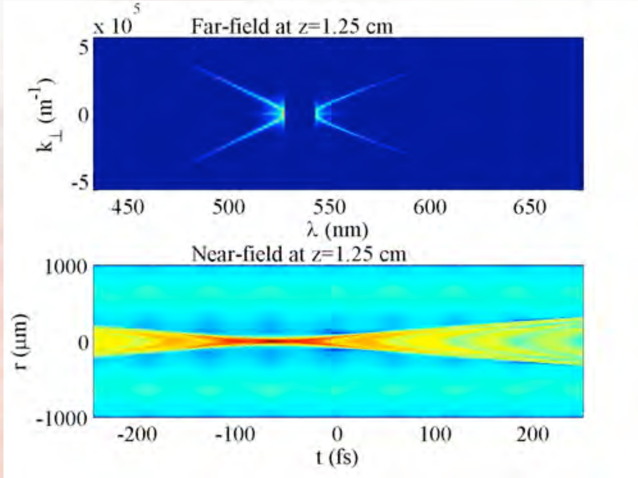
$$F = \frac{1}{2i}\left(E^*\frac{\partial E}{\partial r} - E\frac{\partial E^*}{\partial r}\right)$$

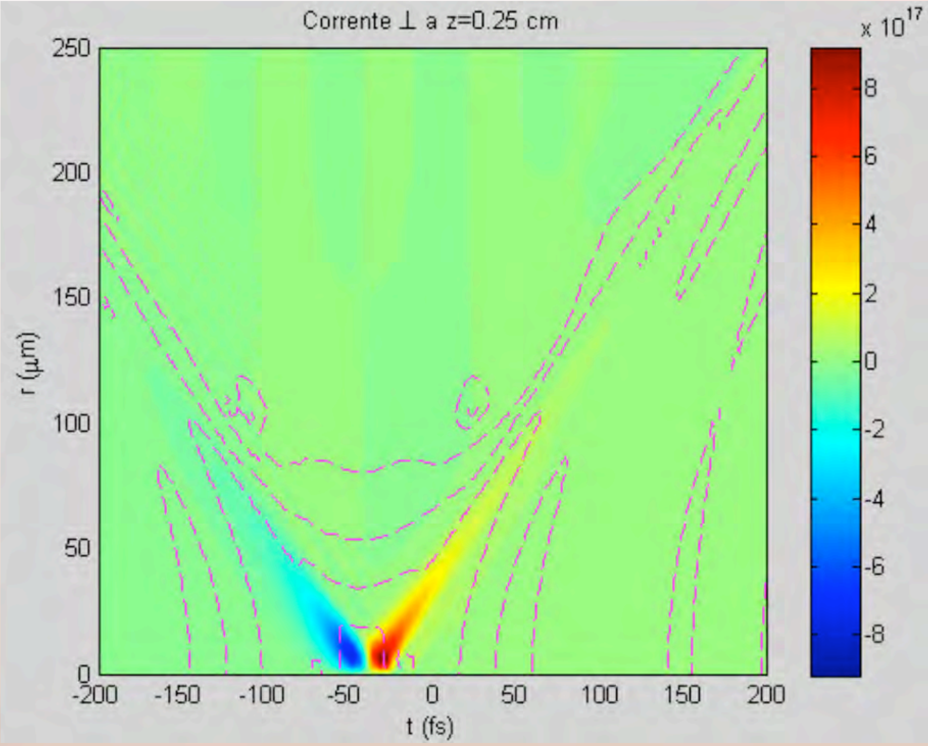


energy flux in an X wave (or Bessel X Wave), characterized in r,t by a zones of incoming energy and others of outgoing energy.

Ultrashort Laser Pulse Diagnostics

linear X wave energy flow.
The figure shows the perp. component of the energy flux for a linear X Wave.

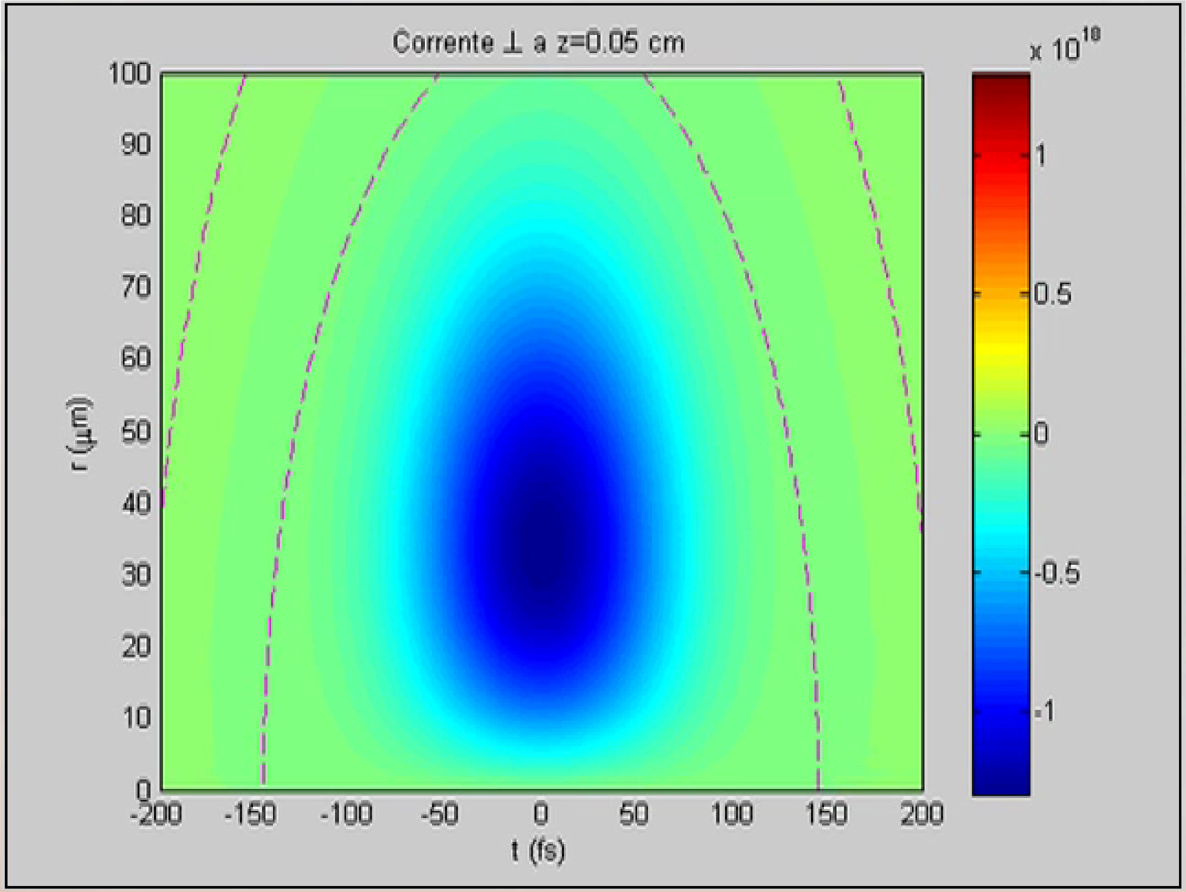


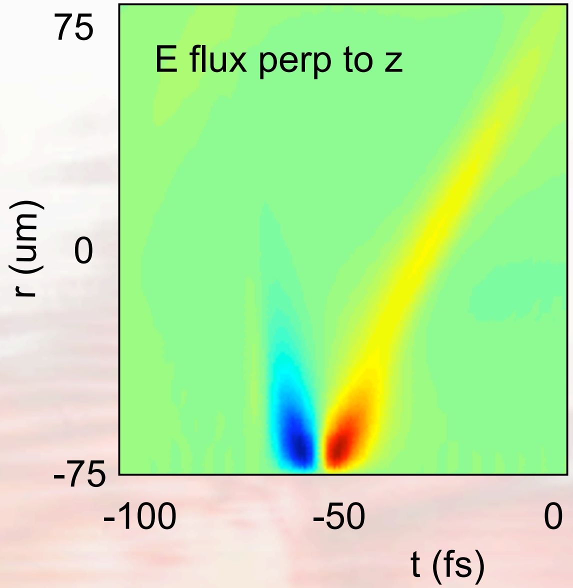


NB: blue = inward flux
red = outward flux

Optical Filaments & X Waves

energy flow in a **filament** in water (800 nm, 4 cm water)

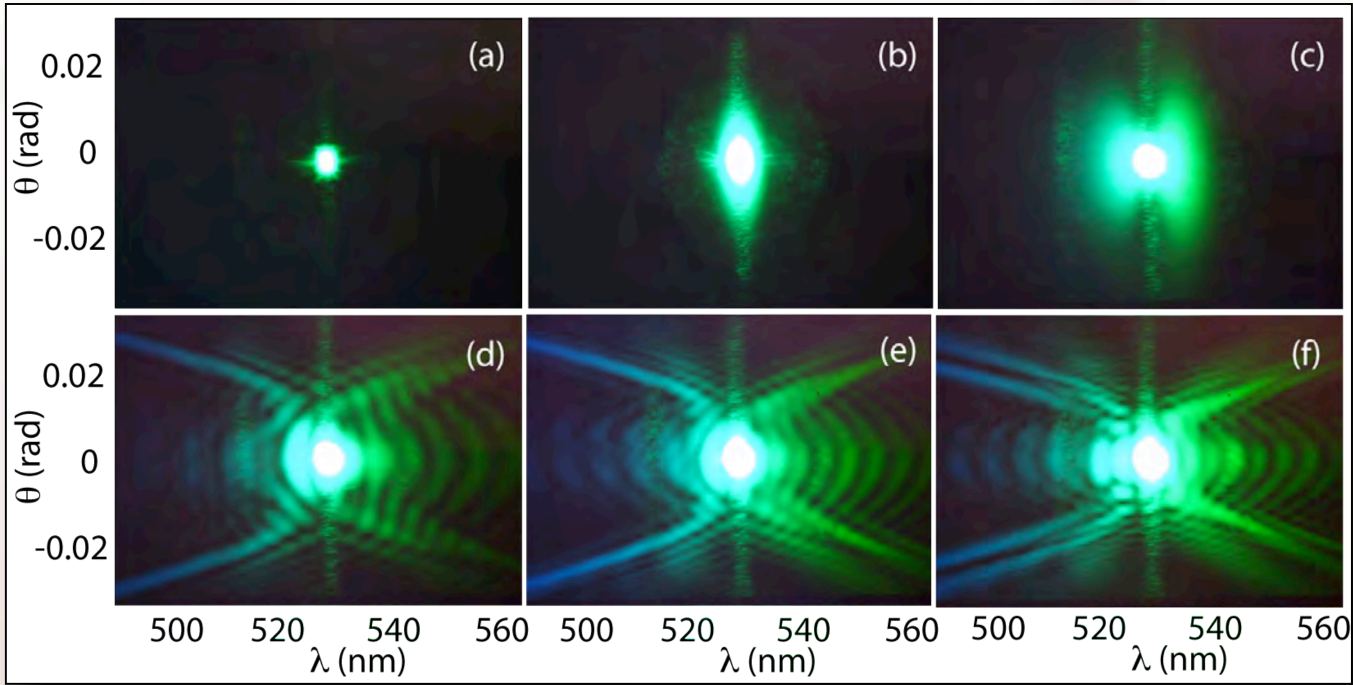




the perp component clearly highlights a
flux identical to that expected for an X Wave with an incoming flux on the leading tails,
ougoing on the trailing tails

Optical Filaments & X Waves

Evidence of X wave formation in filament spectra

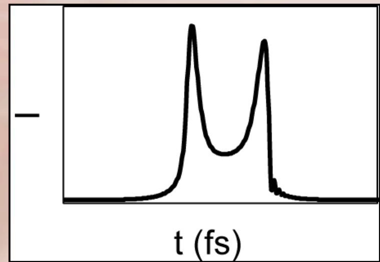


filament spectra, 200 fs, 527 nm pump pulse, focused to 100 μm into 3 cm water

$\theta(\lambda) \rightarrow kz \rightarrow vg = 1/dkz/d\omega$

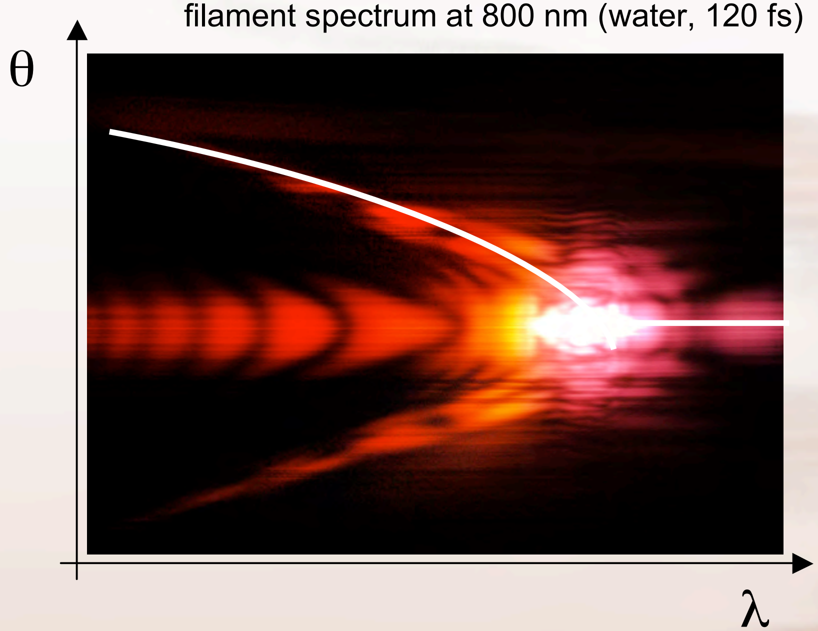
- **kz = const** $\times\ \omega$ (v_g = const) $\rightarrow$ pulses are non-dispersive
  X shaped profiles indicate the formation of (2) ***stationary X waves***

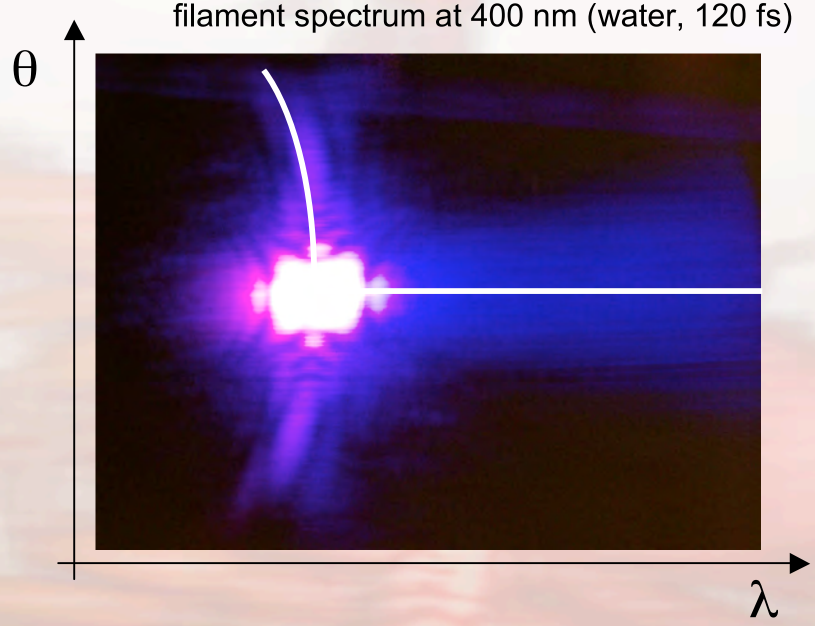
$\rightarrow$ ***pulse splitting***



Optical Filaments & X Waves

Evidence of X wave formation in filament spectra





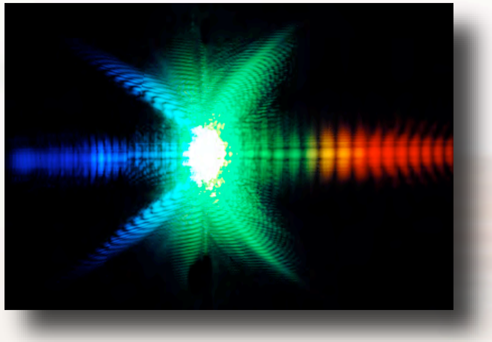
Conical emission spectra at different wavelengths are well reproduced by fitting with X Wave relation

$$
\begin{aligned}
k_\perp &= \sqrt{k^2 - k_z^2} \\
k_z &= (k_0 - \beta) + (k_0' - \alpha)\Omega
\end{aligned}
$$

Optical Filaments & X Waves

Conical emission may be related to X Wave formation...

Question: what is the relation, if any, between Conical and Axial emission?



Models for axial supercontinuum:

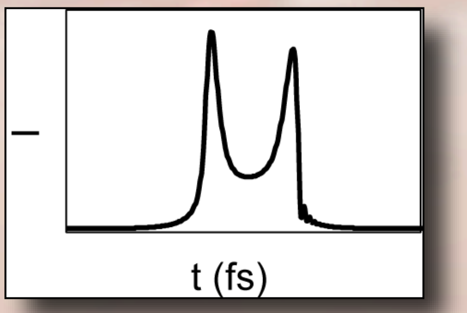
- Shock front formation: SPM enhanced by self-steepening

  "***Steepening occurs on the trailing part of the pulse in materials where the velocity of the peak is slower than that of the wings, beacuse the trailing part of the pulse catches up with the peak***",

  De Martini et al. PRL 1967

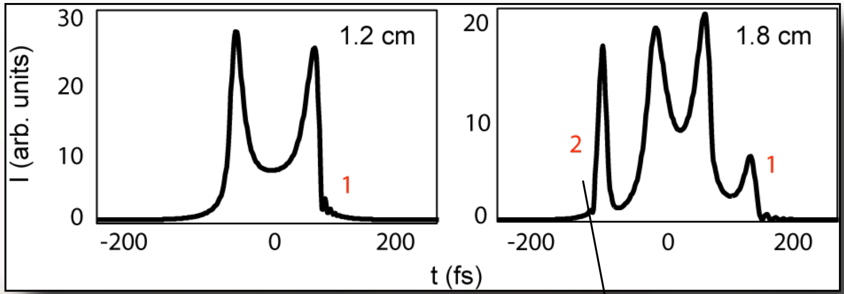
  Dispersion of the Kerr nonlinearity with n2>0 slows down the intense peak

  → only **trailing** shocks are predicted, i.e. **blue** shifted axial supercontinuum

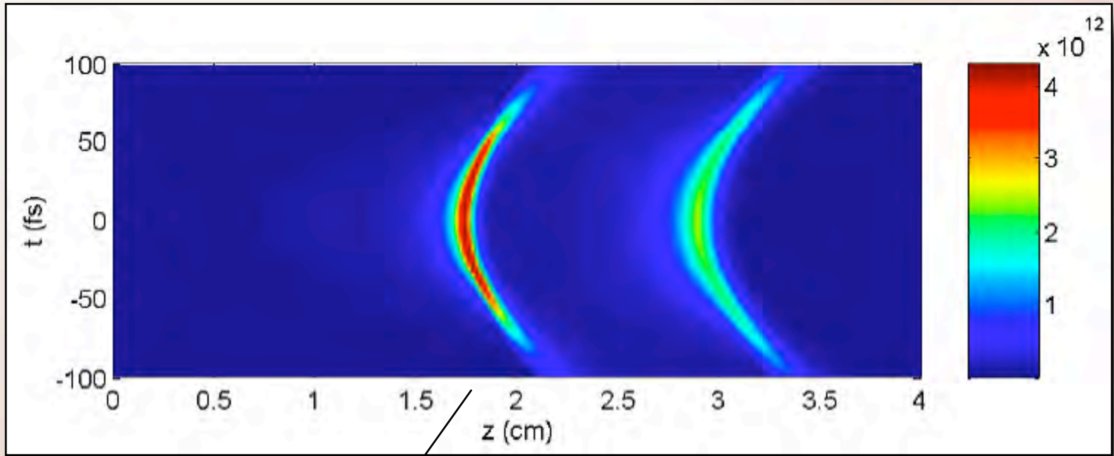


So how can we explain the **red** shifted supercontinuum?

Optical Filaments & X Waves



formation of leading shock front and wave-breaking

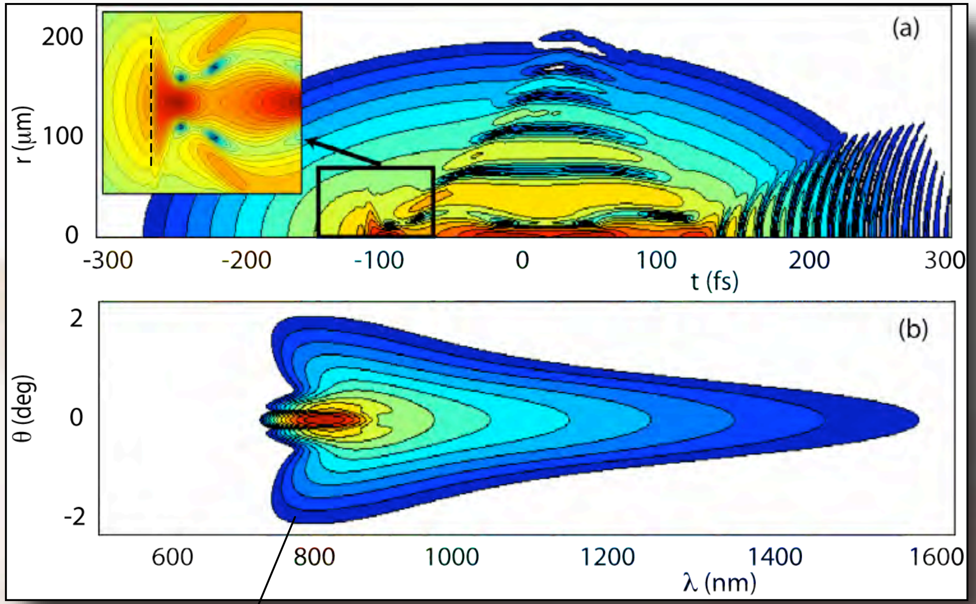


pulse splitting

X wave model:

pulse splitting is the result of the formation of 2 X Waves, 1 super-, 1 sub-luminal

Optical Filaments & X Waves



→ sub-luminal peak + n2>0 forms a trailing shock
→ super-luminal peak + n2>0 forms a ***leading*** shock

Tight connection between Conical and Axial emission:
Conical Emission sustains the formation of sub and super-luminal intensity peaks that travel at a different vg with respect to the surrounding energy background. Kerr nonlinearity then leads to rising or trailing shock formation, i.e. axial super-continuum

Conclusions

X Waves: "intelligent" choice of model pulse for understanding filamentation dynamics
e.g. conical emission, pulse splitting, shock front formation

Current projects:

Raman X Waves: nonlinear filamentation optics
cascaded Raman X formation - phase and group locked pulses
extend studies to gas media

Phase matching: use conical waves for phase-matched frequency conversion, e.g. in the EUV region

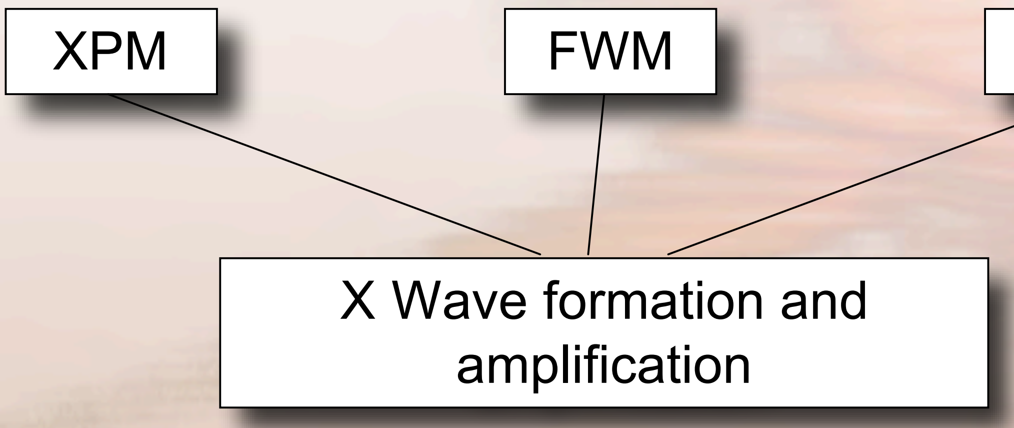
Nonlinear Filamentation Optics

Filaments:
extremely high peak intensities (>TW/cm^2 in condensed media)
pulse intensity peak remains localized over many diffraction lengths

Filaments are the ideal pump source for nonlinear optical interactions

"*Nonlinear Filamentation Optics*", term introduced by S.L. Chin to describe a series of results

- third harmonic generation
- efficient FWM parametric conversion in air (seeded configuration)



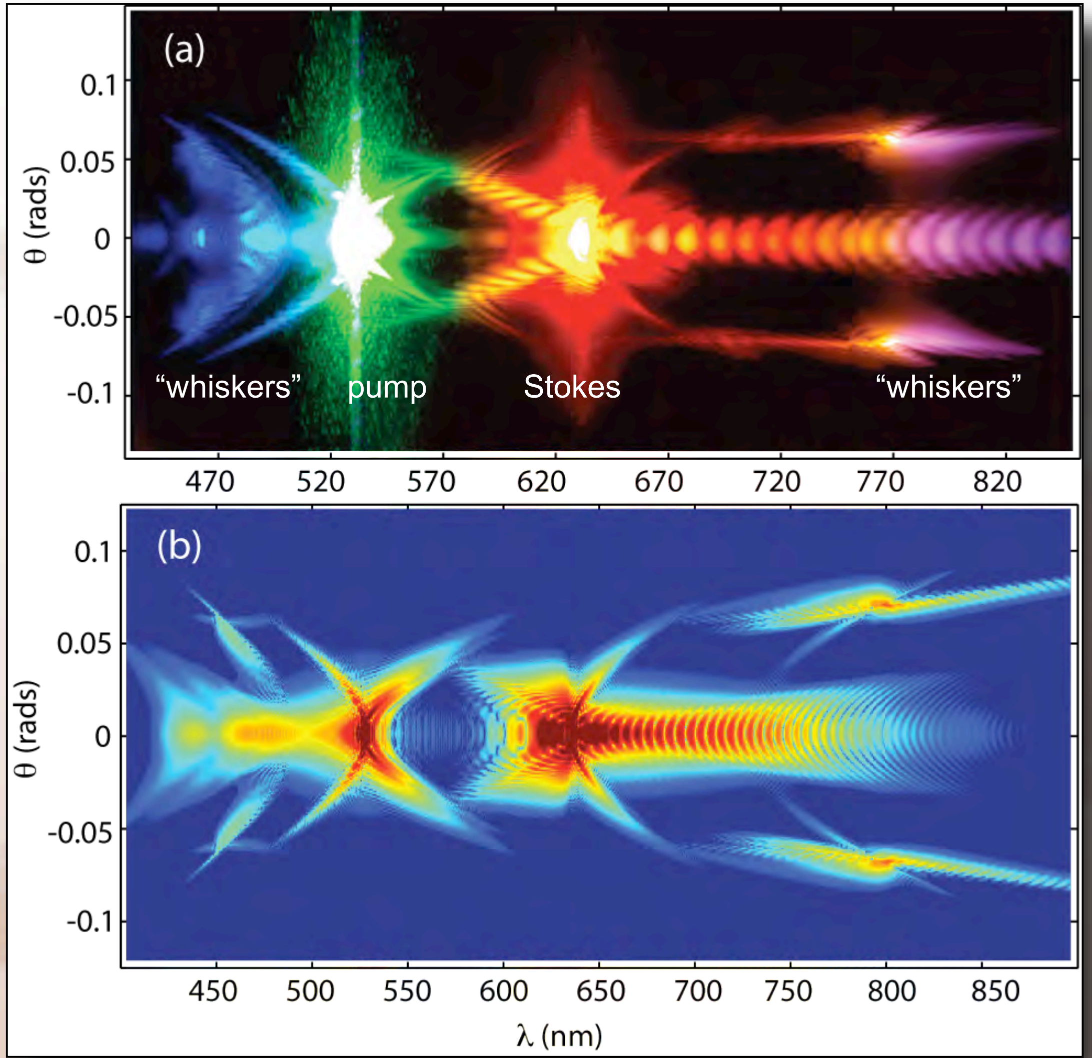
Nonlinear Filamentation Optics

(θ,λ) spectra

Spectra show impressive reshaping

Seed is amplified into X wave
Strong axial components appear
+ interesting off-axis components
at blue (~470 nm) and IR (~800 nm)
→ "whiskers"

All features are well reproduced in
numerical experiments
indicating correct choice of model
and parameters

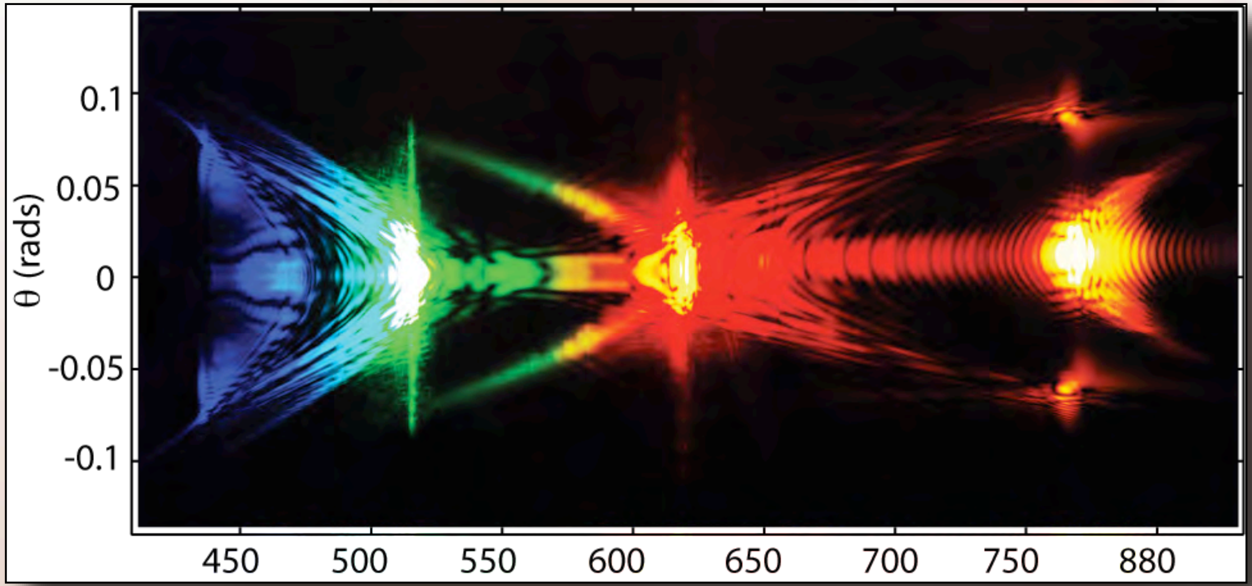


Epump = 3400 nJ

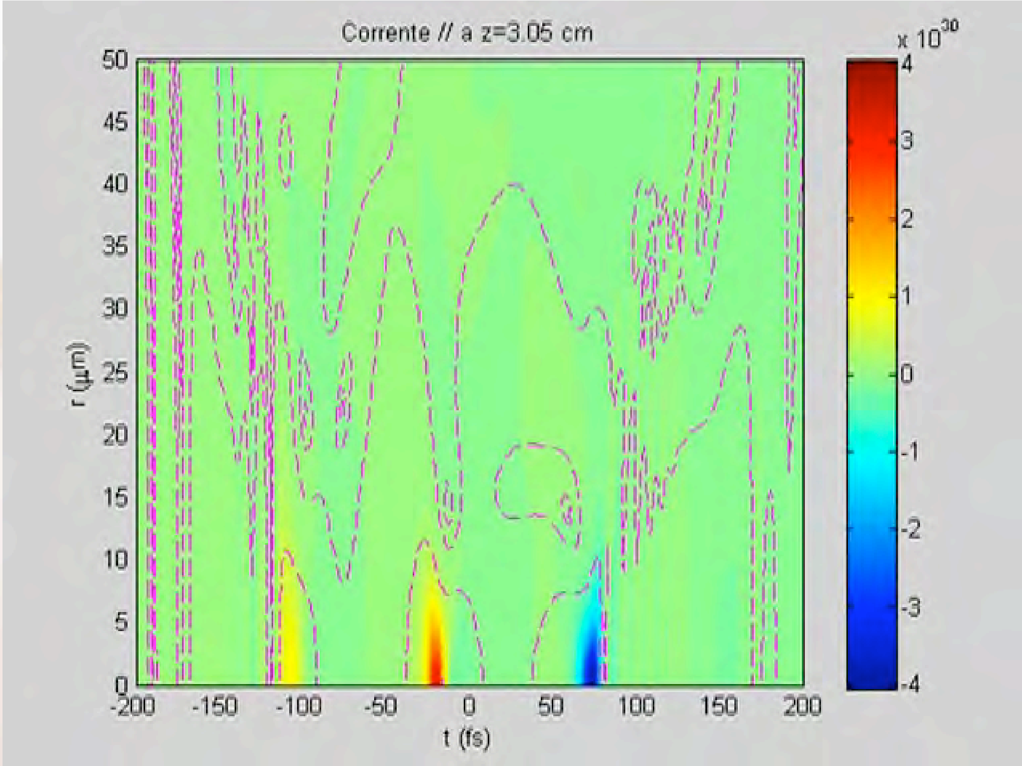
Nonlinear Filamentation Optics

Cascaded Raman X generation:

ethanol: 20x higher gain than water, Raman X (@ 623 nm) becomes dominant feature in the spectrum
@ 800 nm a second-Stokes, cascaded Raman X pulse is generated



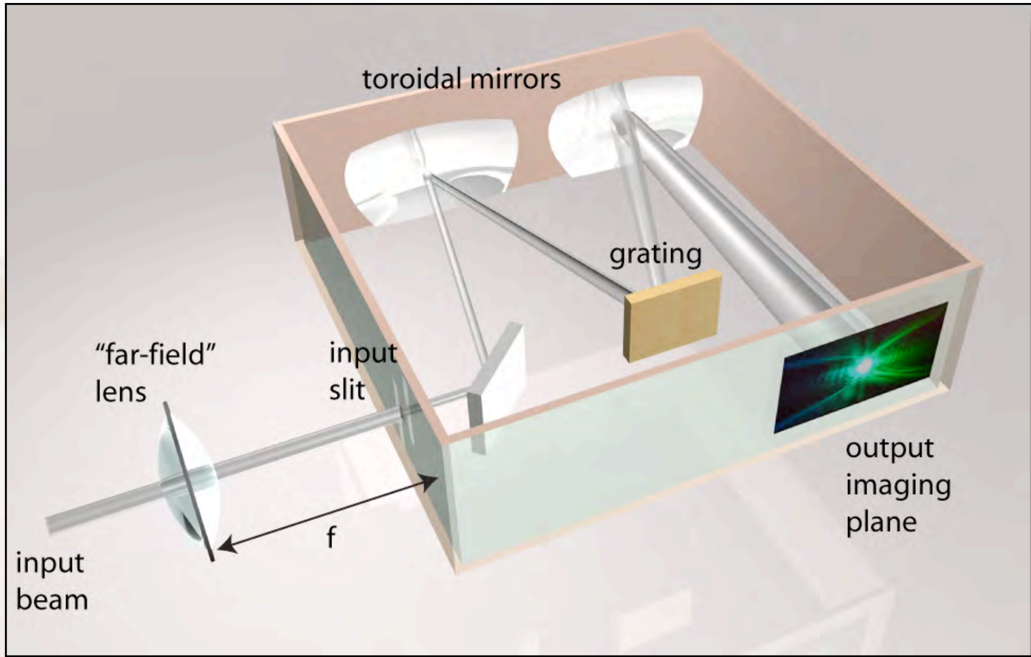
Optical Filaments & X Waves

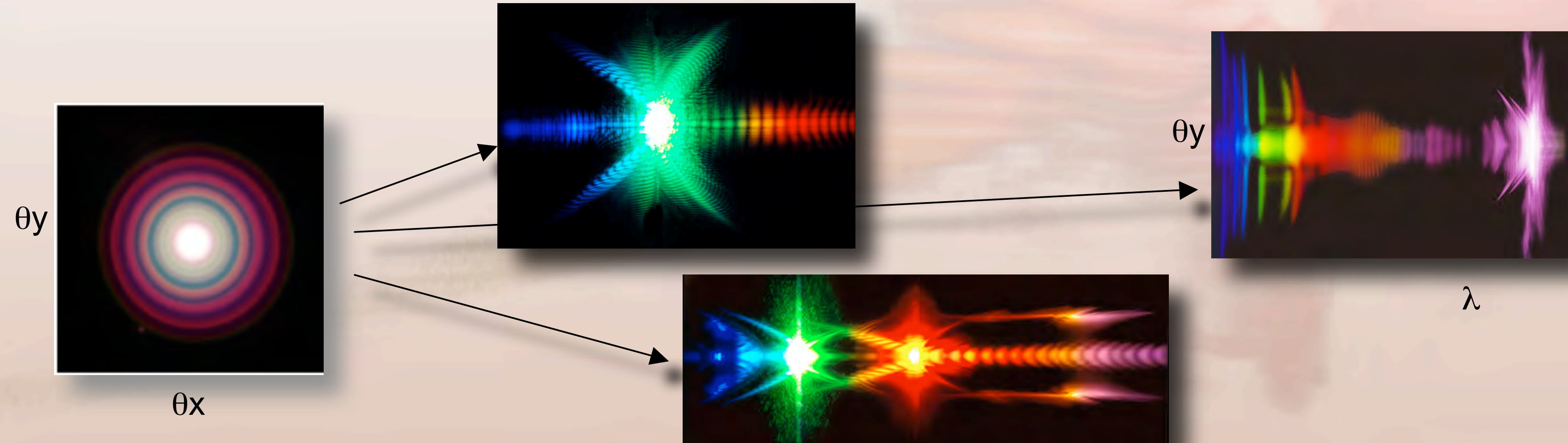


The parallel (to z) energy flow shows a flux direction opposite to the pulse direction

Ultrashort Laser Pulse Diagnostics

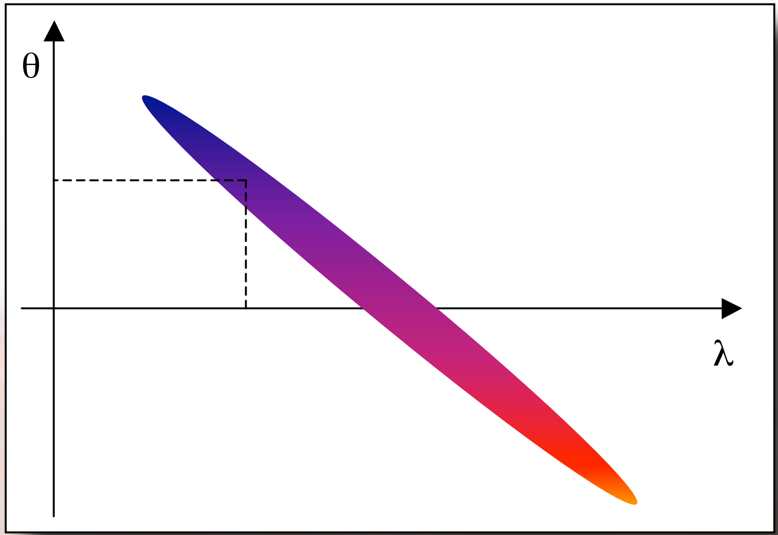
The Imaging Spectrometer





(θ,λ) measurements

Group velocity determination from (θ,λ) spectra...



INGREDIENTS:

spectral profile gives

$$\theta = \theta(\omega)$$

material Sellmeier gives

$$n = n(\omega)$$

$$k_z = \sqrt{k^2 - k_\perp^2} = k\sqrt{1 - \frac{k_\perp^2}{k^2}} = k\sqrt{1-\theta^2}$$

$$v_g = \left(\frac{dk_z}{d\omega}\right)^{-1}$$

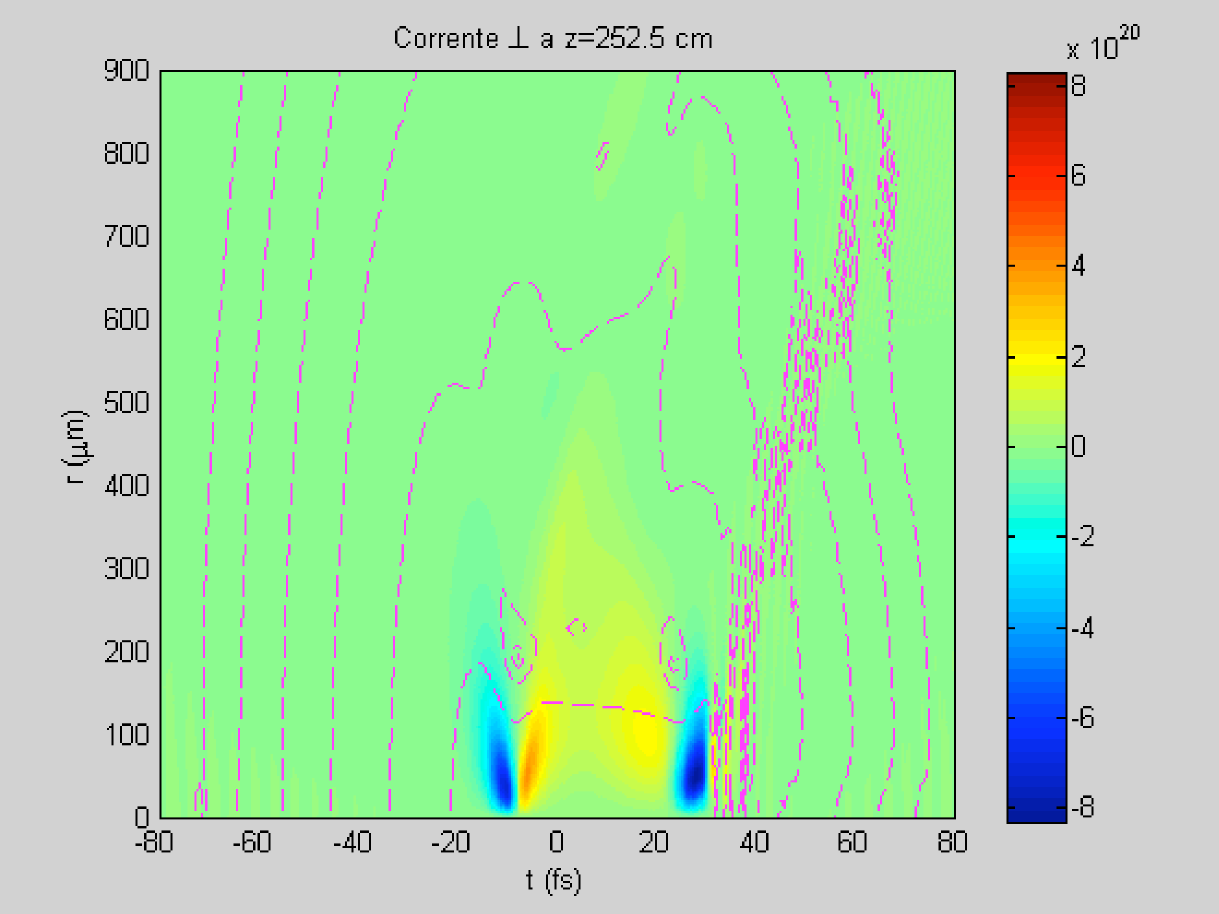
This method works for any pulse shape
- Gaussian spectrum: kz =k, vg is given by material dispersion
- tilted pulse: vg is given by disp. + additional tilt contribution

NB: in general vg=vg(ω) ≠ const
e.g. the Gaussian vg = $dk/d\omega|_{\omega 0}$

IF vg = const, i.e. kz ∝ω
→ the pulse is **non-dispersive**

Ultrashort Laser Pulse Diagnostics

energy flow in a **filament** in air (800 nm)
the film shows propagation from 2.5 to 3 m from the focusing lens (f = 4 m)



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