Discrete Cavity Solitons

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Credits

Most of the work was done by Oleg Egorov

- U. Peschel, O. Egorov, and F. Lederer, ‘Discrete cavity solitons’, Opt. Lett. 29 (2004) 1909
- O. Egorov, U. Peschel, and F. Lederer, 'Mobility of discrete cavity solitons', Phys. Rev. E 72 (2005) 066603
- O. Egorov, U. Peschel, and F. Lederer, 'Discrete quadratic cavity solitons’ Phys. Rev. E 71 (2005) 056612
- O. Egorov and F. Lederer, and K. Staliunas, ‘Sub-diffractive discrete cavity solitons’, Opt. Lett. (to appear August 1)
- O. Egorov and F. Lederer, and Y. S. Kivshar,’ How does an inclined holding beam affect discrete modulational instability and soliton formation in arrays of nonlinear cavities? ‘ Opt. Exp. 15 (07) 4149

The work was supported by a grant of the Deutsche Forschungs-gemeinschaft, Forschergruppe 532.

Credits - Collaborations

- U. Peschel, O. Egorov, and F. Lederer, ‘Discrete cavity solitons’, Opt. Lett. 29 (2004) 1909
- O. Egorov, U. Peschel, and F. Lederer, 'Mobility of discrete cavity solitons', Phys. Rev. E 72 (2005) 066603
- O. Egorov, U. Peschel, and F. Lederer, 'Discrete quadratic cavity solitons’ Phys. Rev. E 71 (2005) 056612
- O. Egorov and F. Lederer, and K. Staliunas, ‘Sub-diffractive discrete cavity solitons’, Opt. Lett. (to appear August 1)
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Outline

1. Introduction
2. Resting Discrete Cavity Solitons (DCS)
3. Mobility of Resting DCS vs. Moving DCS
4. Sub-diffractive DCS

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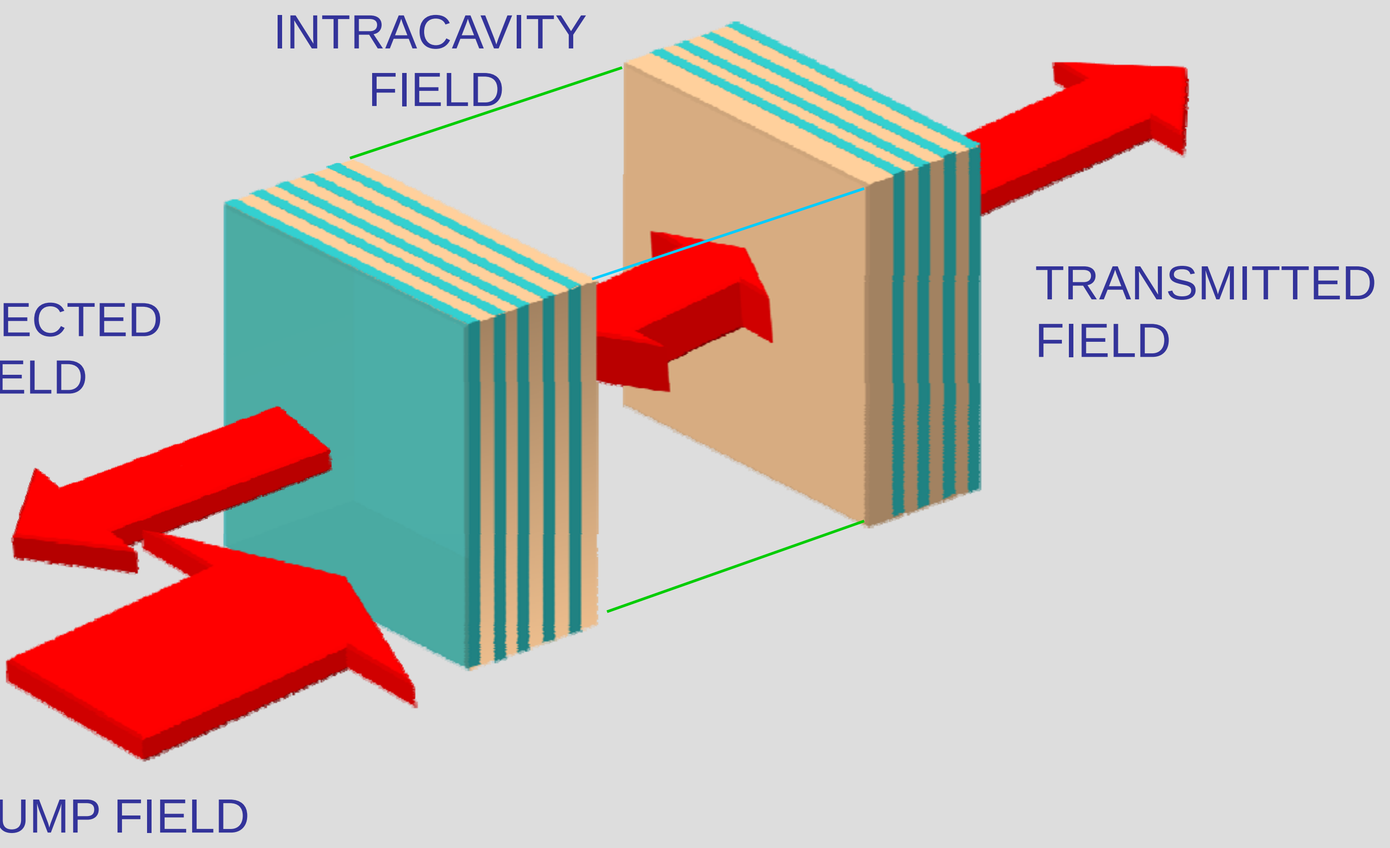
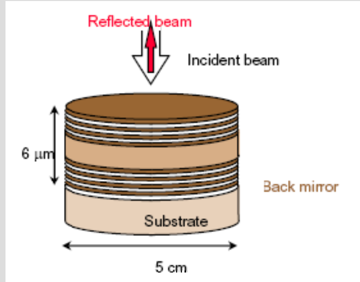
Cavity Solitons

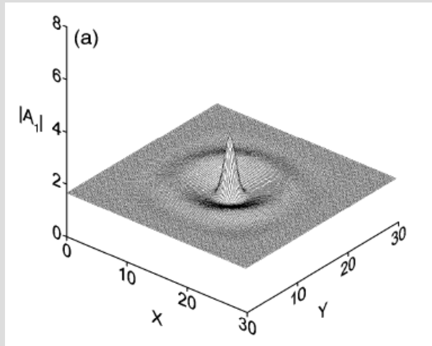


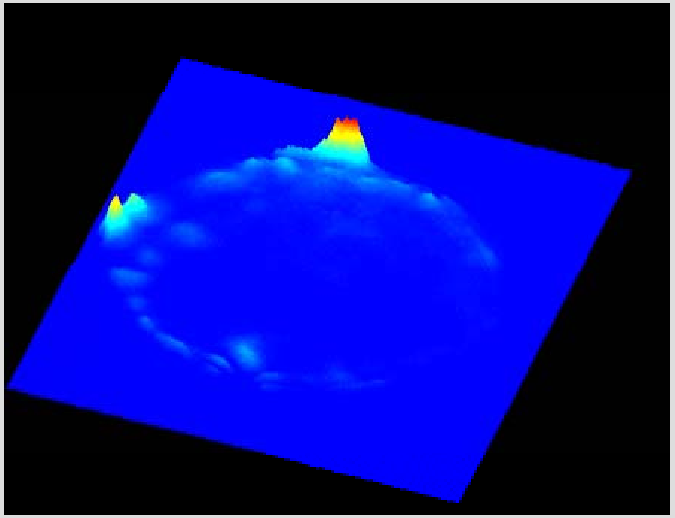
Cavity Solitons

wide aperture passive resonators



$$\left[i\frac{\partial}{\partial t} + \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + i\alpha + \Delta + \chi_{\mathrm{NL}}\left(|u|^2\right) \right] u(t,x) = u_{\mathrm{in}}(t,x)$$





F. Lederer, et al.

Phys. Rev. A56, R3366 (1998)

L. Lugiato et al.

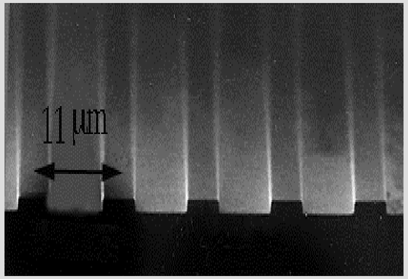
Phys. Rev. Lett. 79, 2042 (1998)

EU project FUNFACS

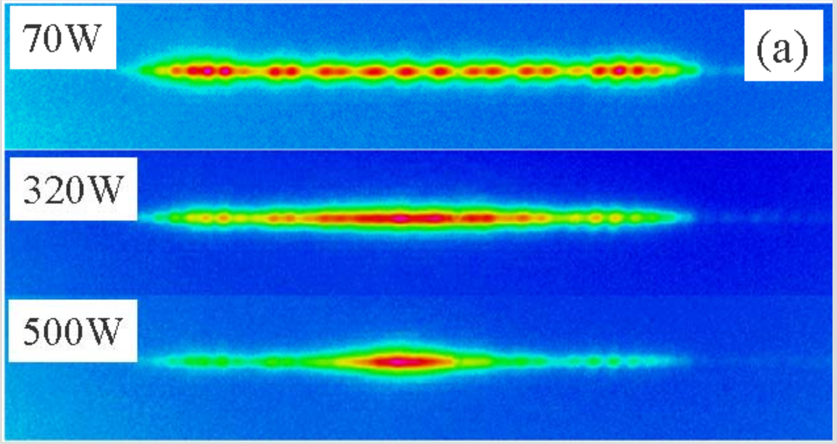
J. Tredicce, M. Giudici, Univ. Nice

Discrete Solitons

waveguide array



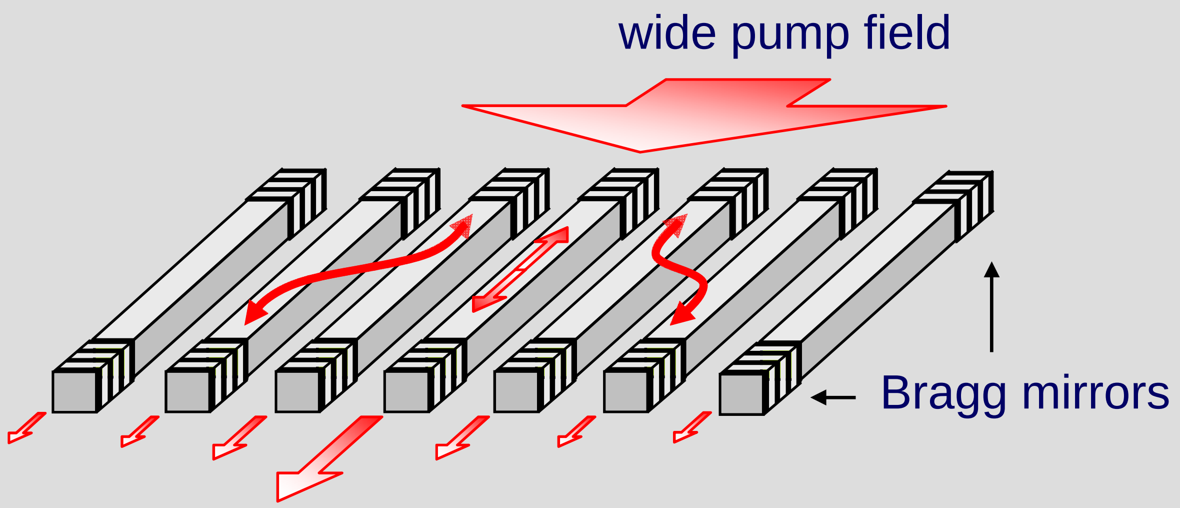
$$i\frac{\partial u_n}{\partial z} + C\left(u_{n+1} + u_{n-1}\right) + \gamma \left|u_n\right|^2 u_n = 0$$



D. N. Christodoulides and R. I. Joseph,
Opt. Lett. 13 (1988) 794

Y. Silberberg et al., Phys. Rev.Lett. 91(98) 3383

Discrete Cavity Solitons



array of nonlinear waveguides

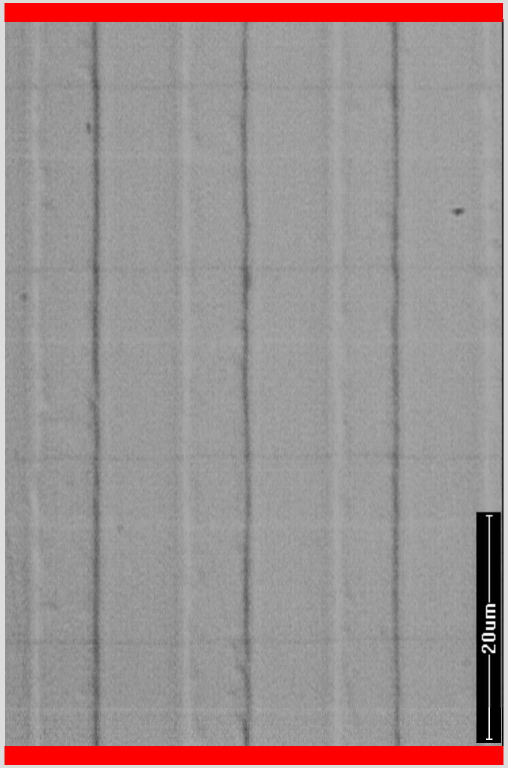
+ mirrors → feedback and radiation losses

+ pump field → energy supply

Array of Nonlinear Waveguide Resonators

PPLN
Periodically Poled
Lithium Niobate

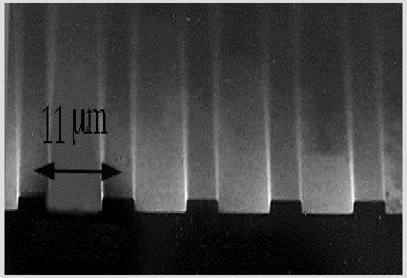
Bragg mirrors



fast quadratic nonlinearity

W. Sohler, Paderborn

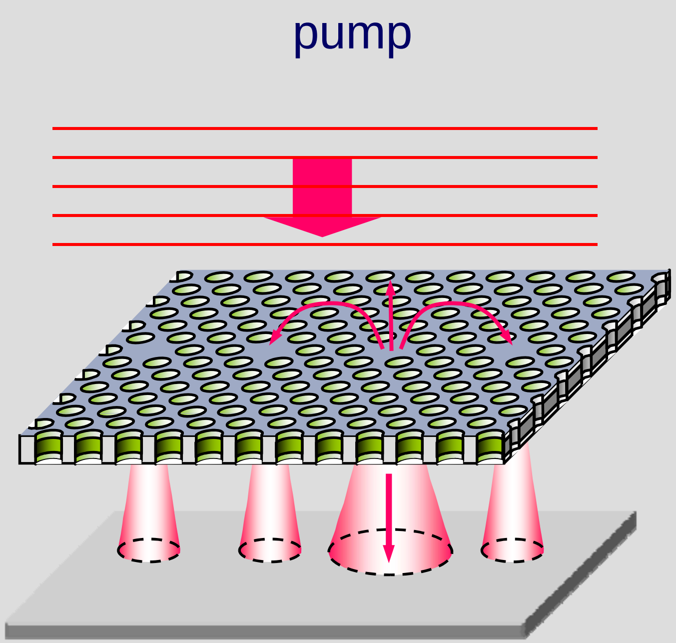
AlGaAs below
half band gap



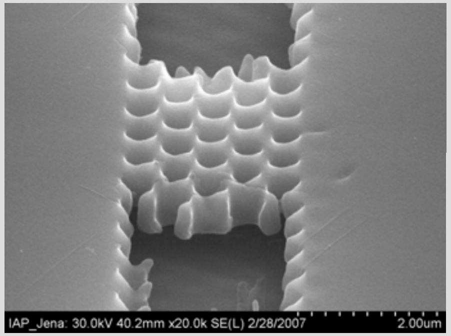
fast cubic nonlinearity

I.S.Aitchison,Toronto

Array of Coupled Defects in PCs

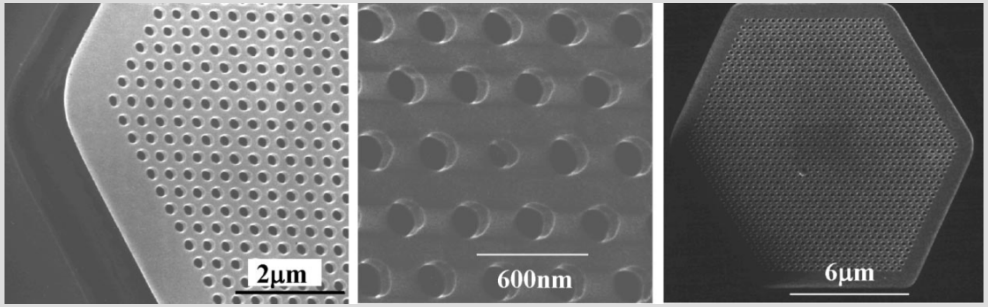


Lithium Niobate – quadratic NL



H. Hartung, E.-B. Kley, A.Tünnermann, W. Wesch, FSU Jena

Semiconductor – cubic NL



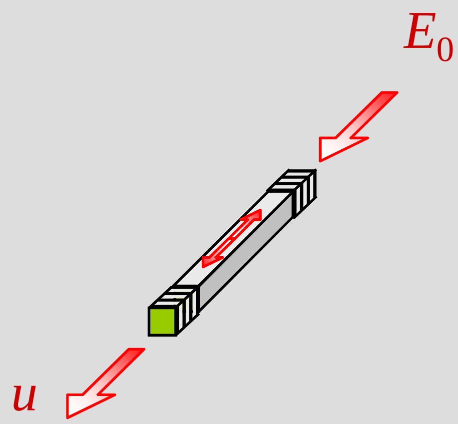
A. Scherer et al.
Appl. Phys. Lett 79, 4289 (2001)

Single Cavity Response

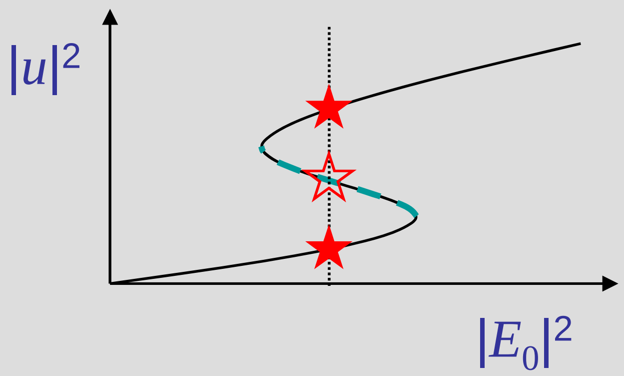
mean-field model for the single cavity

$$i\frac{\partial u}{\partial T}+(i+\Delta)u+\gamma|u|^2u=E_0$$

losses detuning — Kerr NL — pump



bistability

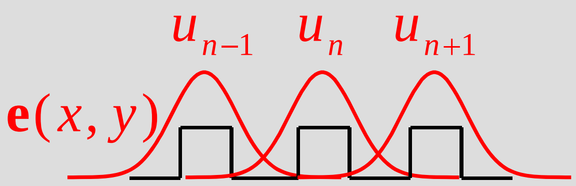


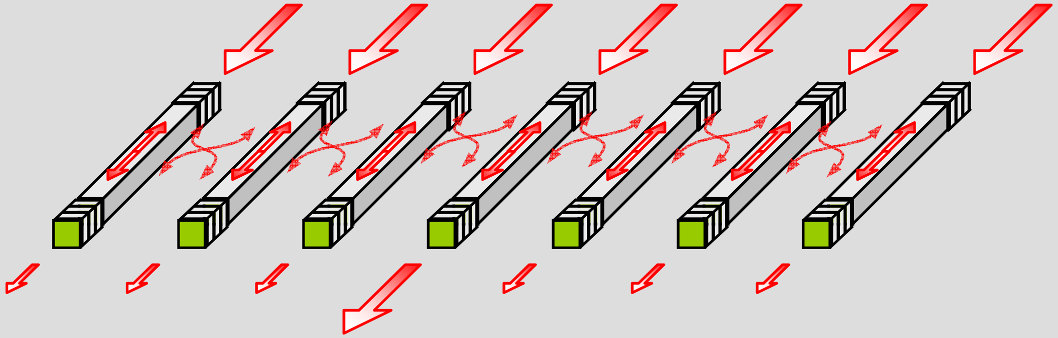
← stable
← unstable
← stable

3 solutions for $\Delta\cdot\gamma<-\sqrt{3}$

Array of Coupled Cavities

evanescent coupling





Array of Coupled Cavities

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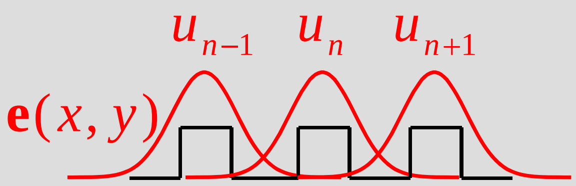
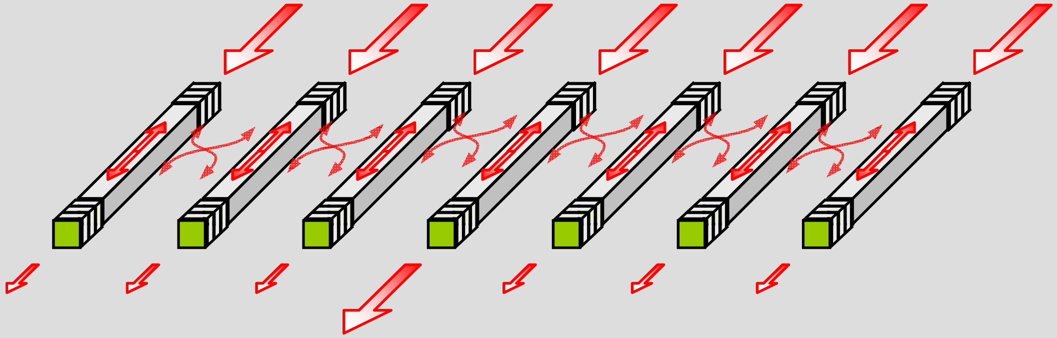




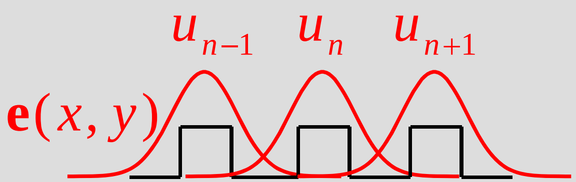
Mean-field and tight-binding approximations

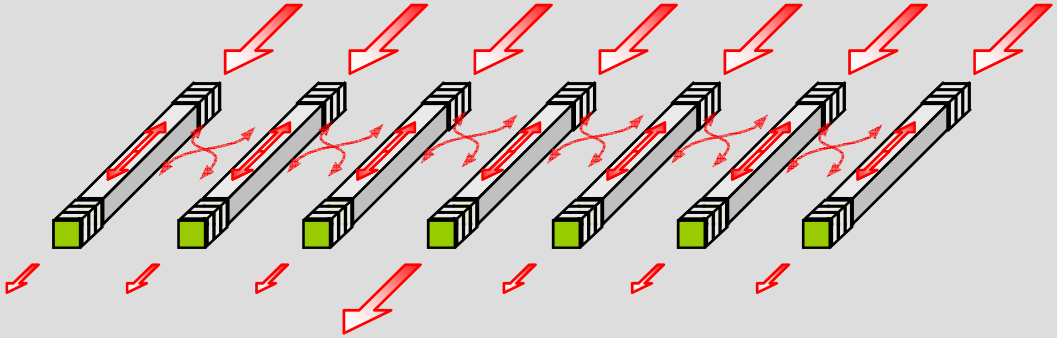
$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i+\Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0$$

coupling or
discrete diffraction

Array of Coupled Cavities - Limits

evanescent coupling





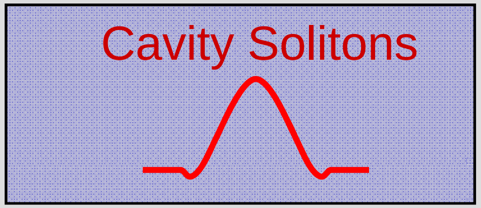
Mean-field and tight-binding approximations

$$i\frac{\partial u(x)}{\partial T} + D\frac{\partial^2}{\partial x^2}u(x) + (i + \Delta)u(x) + \gamma\left|u\right|^2 u(x) = E_0$$

'ordinary' continuous model for wide beams and normal incidence

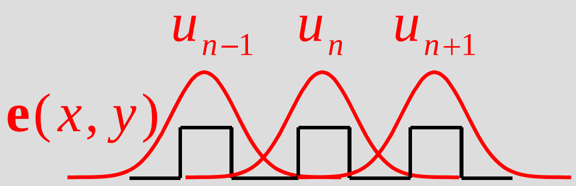
"continuous" limit

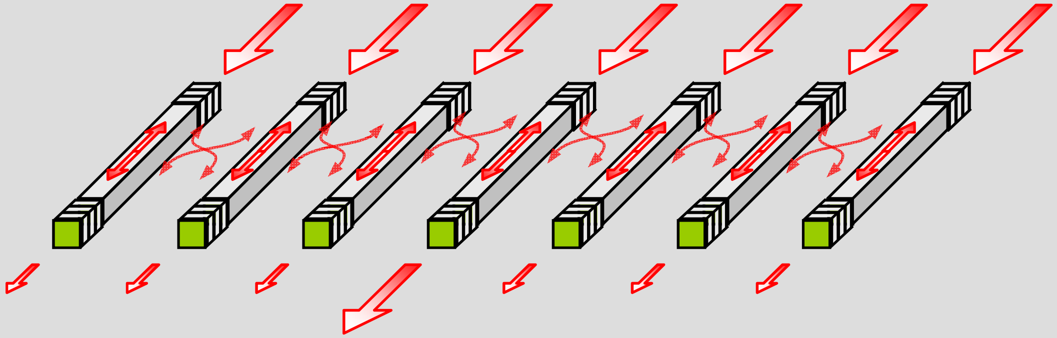
$C \to \infty$



Array of Coupled Cavities - Limits

evanescent coupling



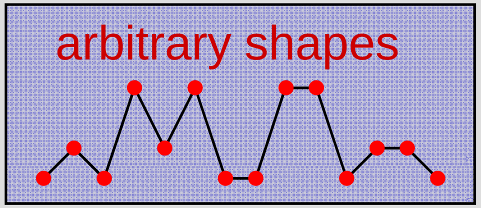


Mean-field and tight-binding approximations

$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i+\Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0$$

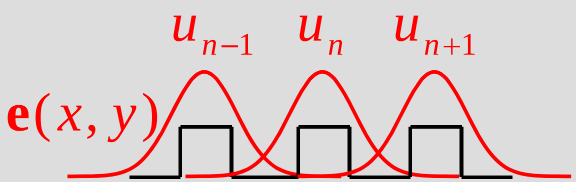
"anti-continuous" limit

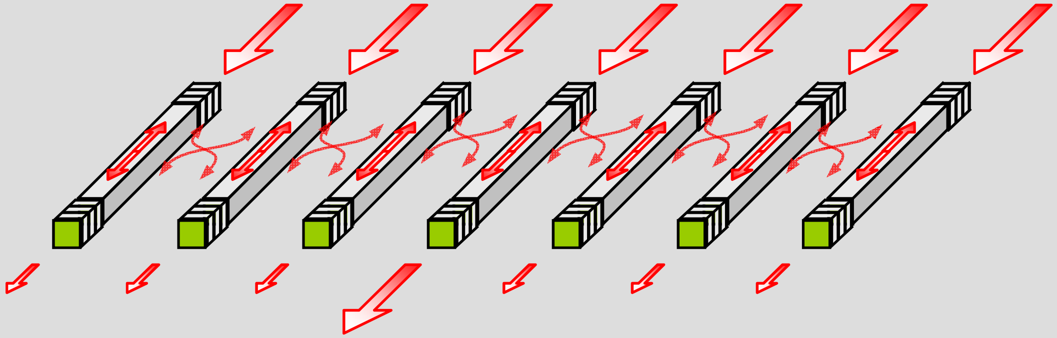
$C=0$



Array of Coupled Cavities

evanescent coupling





Mean-field and tight-binding approximations

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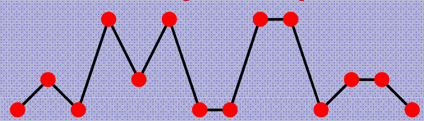
"anti-continuous" limit

C=0

"continuous" limit

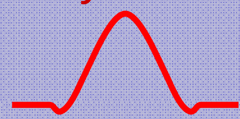
$C \to \infty$

arbitrary shapes

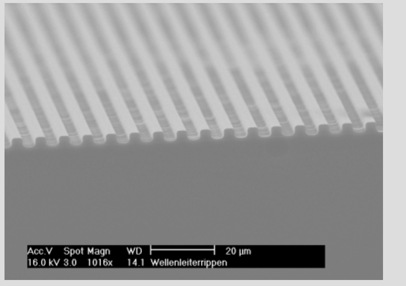


Discrete Cavity Solitons

Cavity Solitons



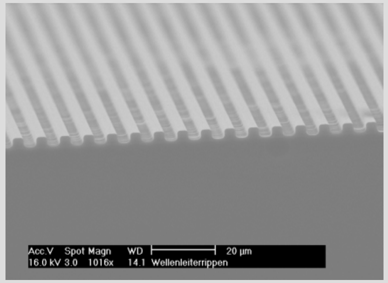
Discrete Diffraction



1D waveguide array

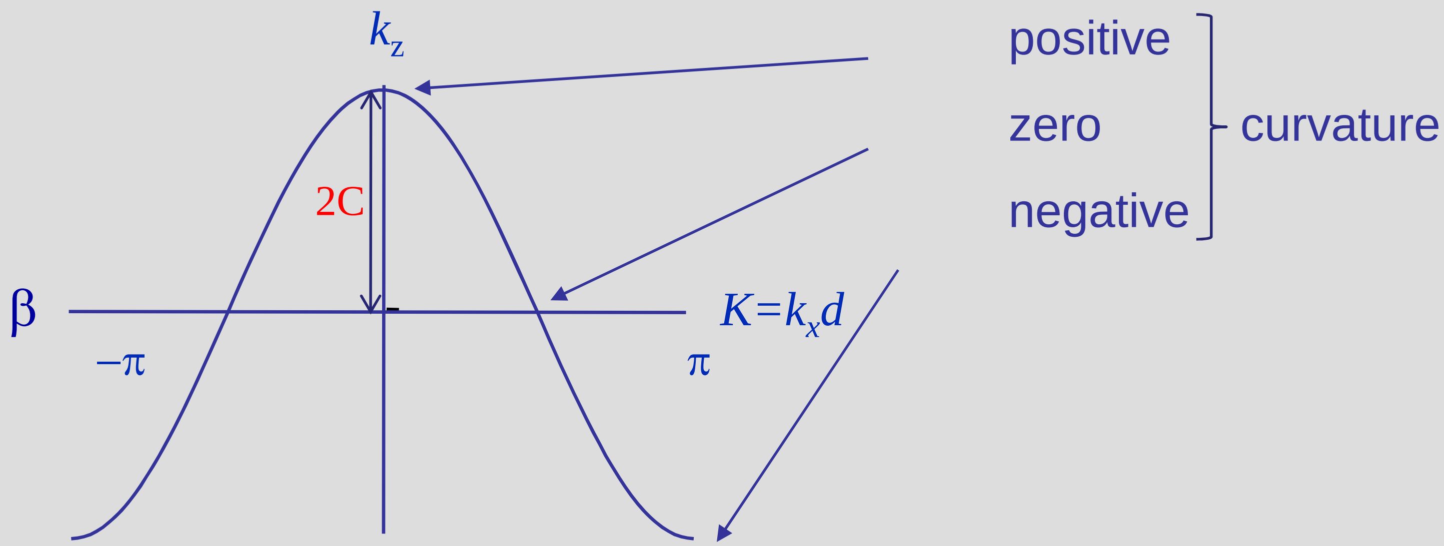
NM: Bloch waves → DR: $k_z = \beta + 2C(\omega)\cos(k_x d)$

Discrete Diffraction



1D waveguide array

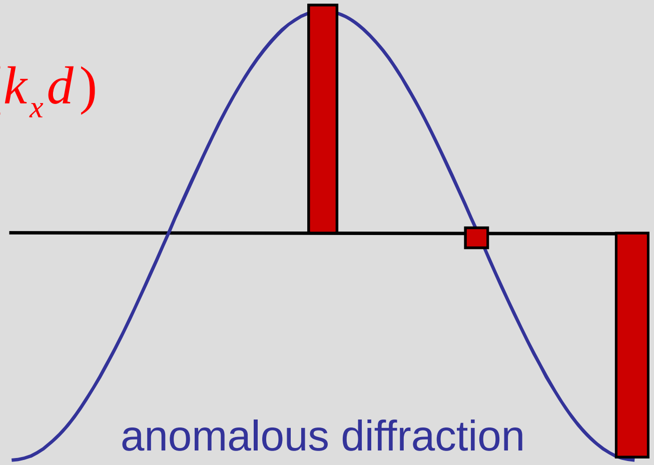
NM: Bloch waves → DR: $k_z = \beta + 2C(\omega)\cos(k_x d)$

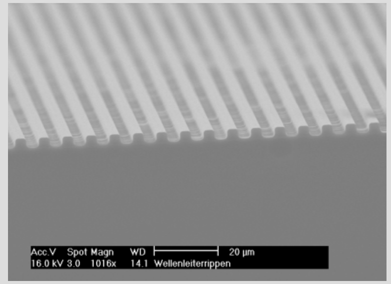


Discrete Diffraction

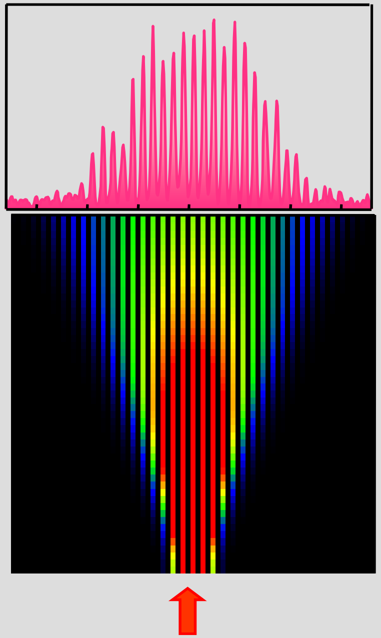
T. Pertsch, U. Peschel, and F. Lederer,
Phys. Rev. Lett., **88** (02) 093901

$$k_z = \beta + 2C(\omega)\cos(k_x d)$$

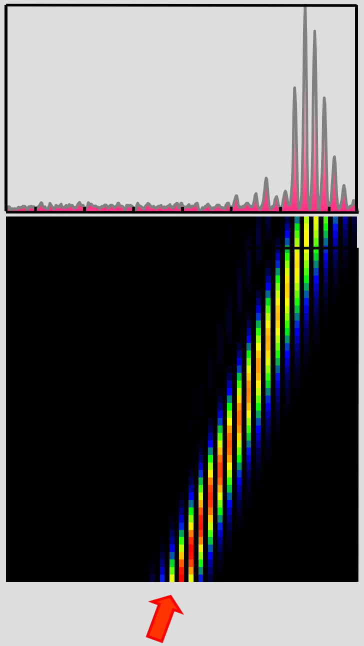




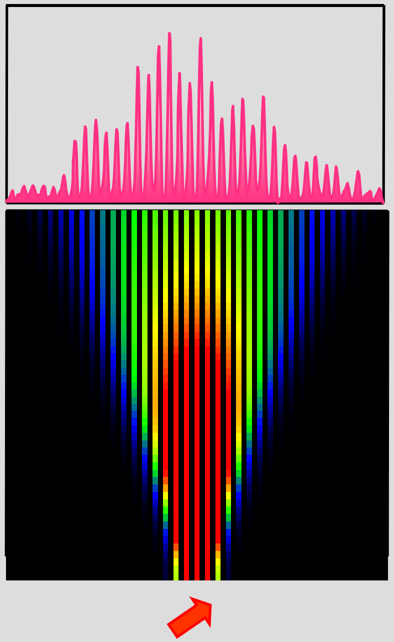
canonical



anomalous diffraction



anom. refraction and diffraction

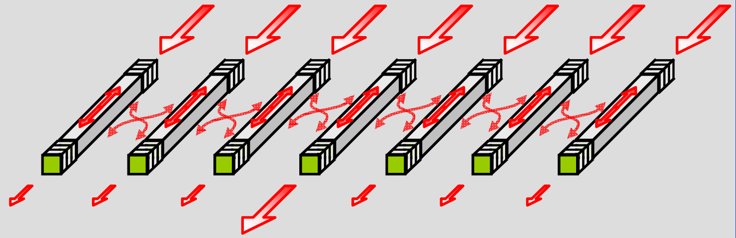


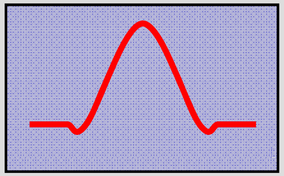
Outline

1. Introduction
2. Resting Discrete Cavity Solitons (DCS)
3. Mobility of Resting DCS vs. Moving DCS
4. Sub-diffractive DCS

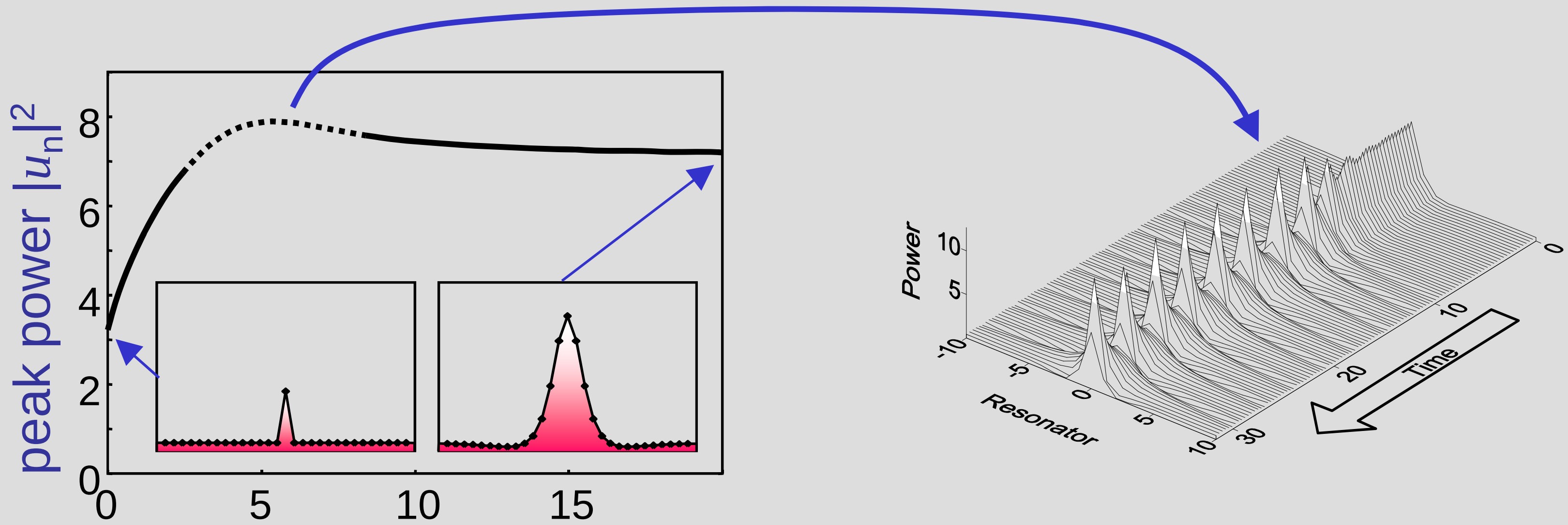
Discrete Cavity Soliton – Simplest Solutions

1) focusing nonlinearity





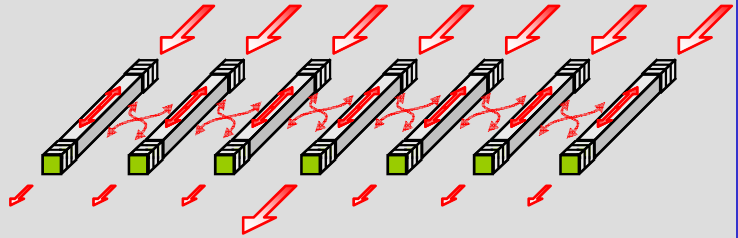
→ bright solitons in continuous limit



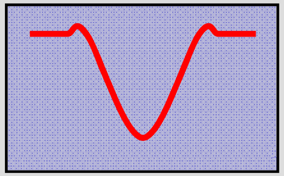
Hopf instability

→ coupling (discrete diffraction) controls soliton formation

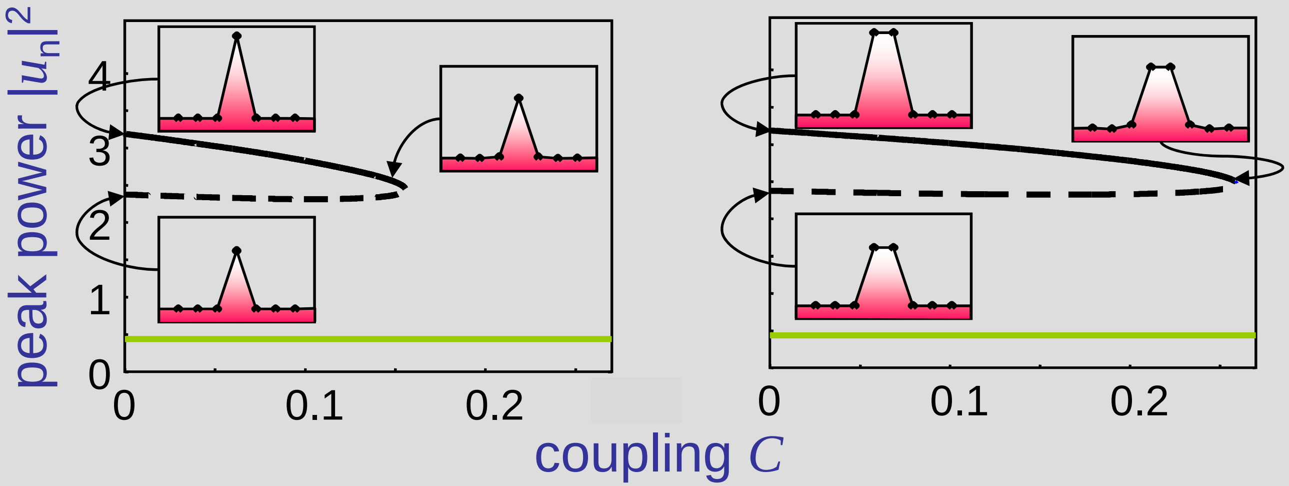
Discrete Cavity Soliton – Simplest Solutions



2) defocusing nonlinearity

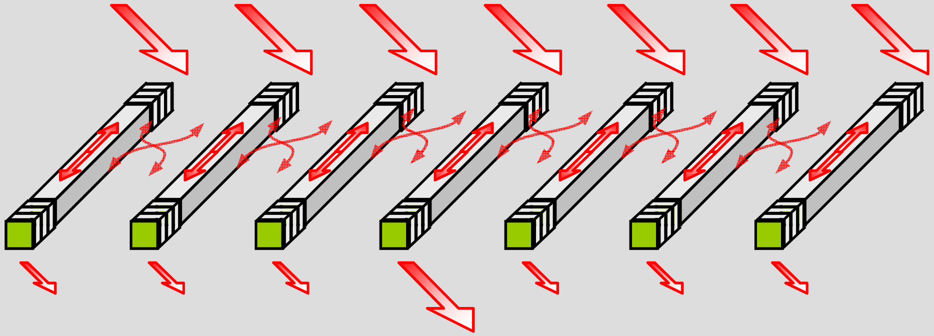


→ dark solitons in continuous limit



→ bright solitons disappear for large coupling

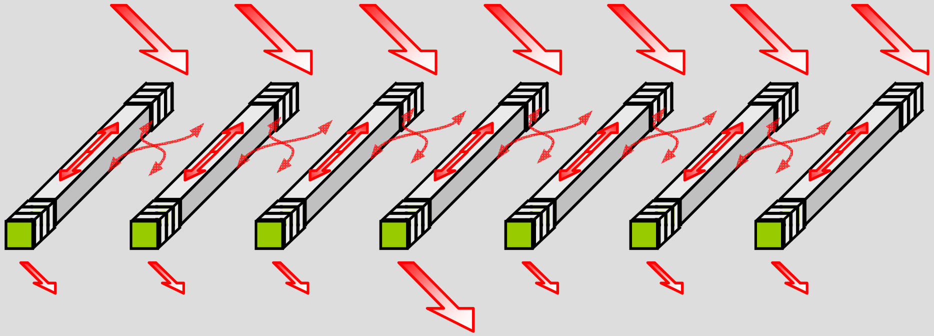
Effect of an Inclined Holding Beam



inclined holding beam

$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i + \Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0 e^{iqn}$$

Effect of an Inclined Holding Beam



inclined holding beam

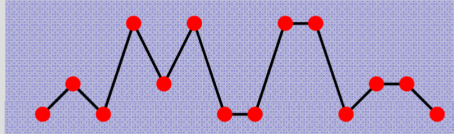
$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i + \Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0 e^{iqn}$$

"anti-continuous" limit

$C = 0$

no power transfer;

$\rightarrow$ at rest



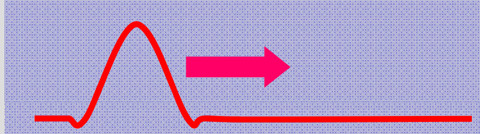
- $q < q_{crit} \rightarrow$ at rest
- $q > q_{crit} \rightarrow$ motion

"continuous" limit

$C \rightarrow \infty$

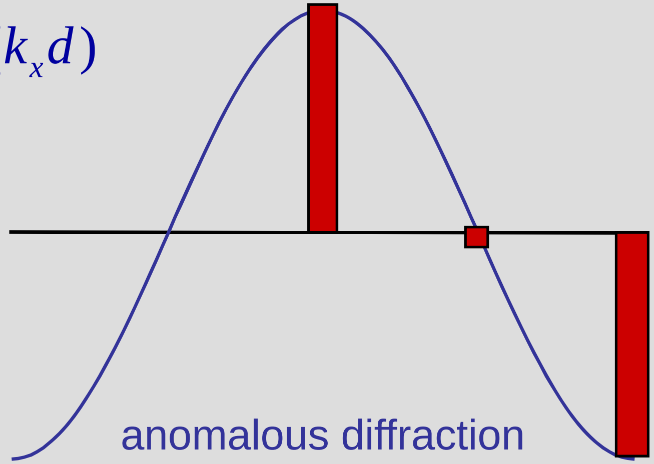
translational symmetry;

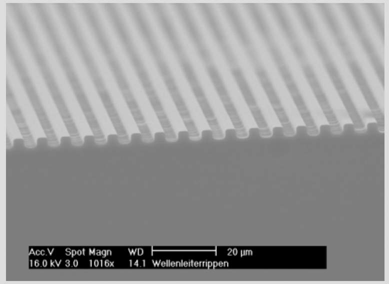
$\rightarrow$ motion



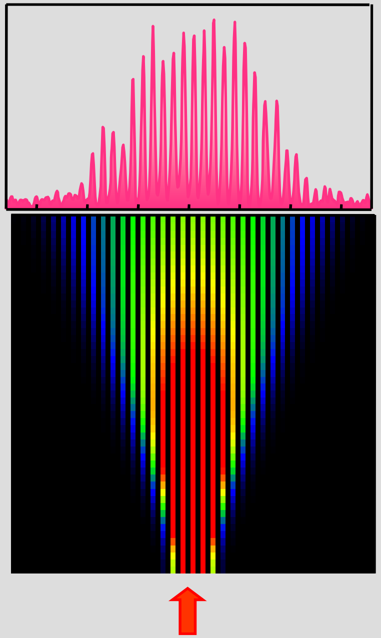
Reminder - Discrete Diffraction

$$k_z = \beta + 2C(\omega)\cos(k_x d) = \beta + 2C\cos(q)$$

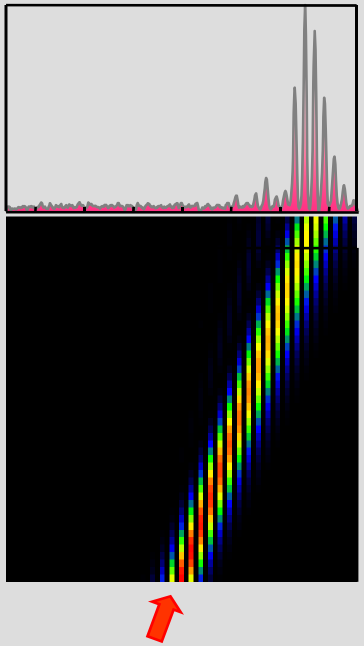




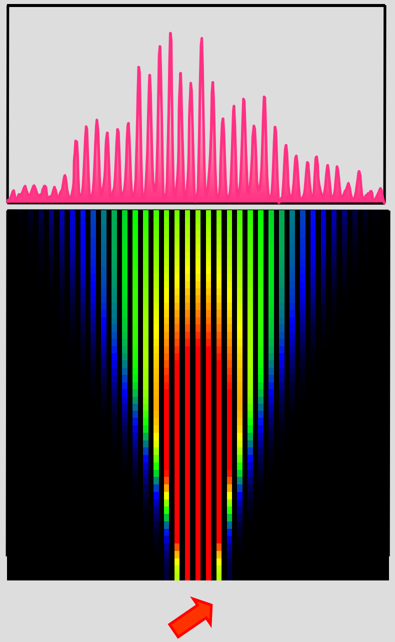
canonical



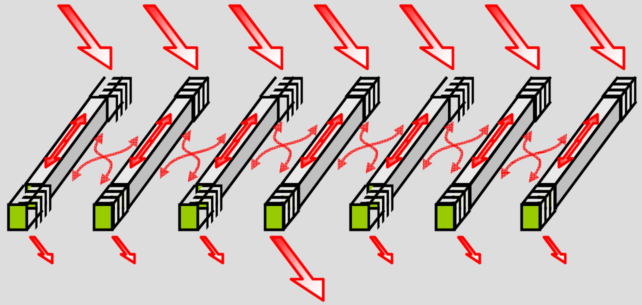
anomalous diffraction



anom. refraction and diffraction



Stability of Discrete Plane Waves



Discrete model

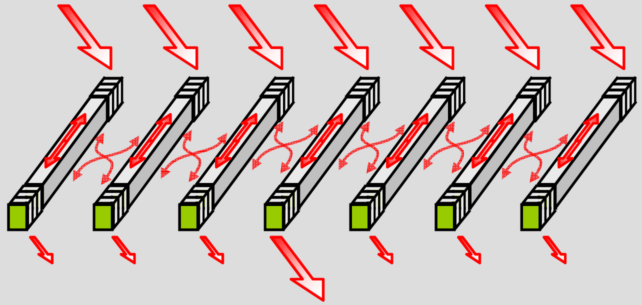
$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i+\Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0 e^{iqn}$$

cw plane wave solution

$$u_n = b\,\mathrm{e}^{iqn}$$

O. Egorov and F. Lederer,
and Y. S. Kivshar,
Opt. Exp. 15 (07) 4149

Stability of discrete plane waves



Discrete model

$$i\frac{\partial u_n}{\partial T} + C\left(u_{n+1} + u_{n-1} - 2u_n\right) + (i+\Delta)u_n + \gamma\left|u_n\right|^2 u_n = E_0 e^{iqn}$$

perturbed plane wave

$$u_n = \left(b + \underbrace{a\exp\left(\lambda T + iQn\right)}_{\text{perturbation}}\right)e^{iqn}$$

bistability condition

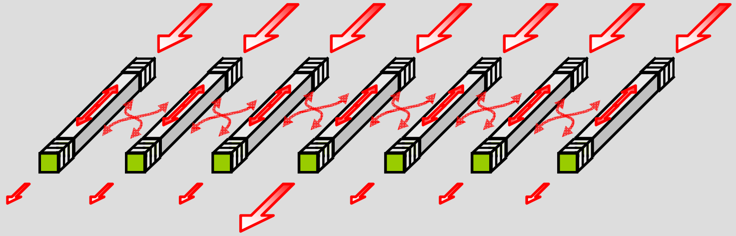
$$\gamma\left[\Delta + 2C\left(\cos q - 1\right)\right] < -\sqrt{3}$$

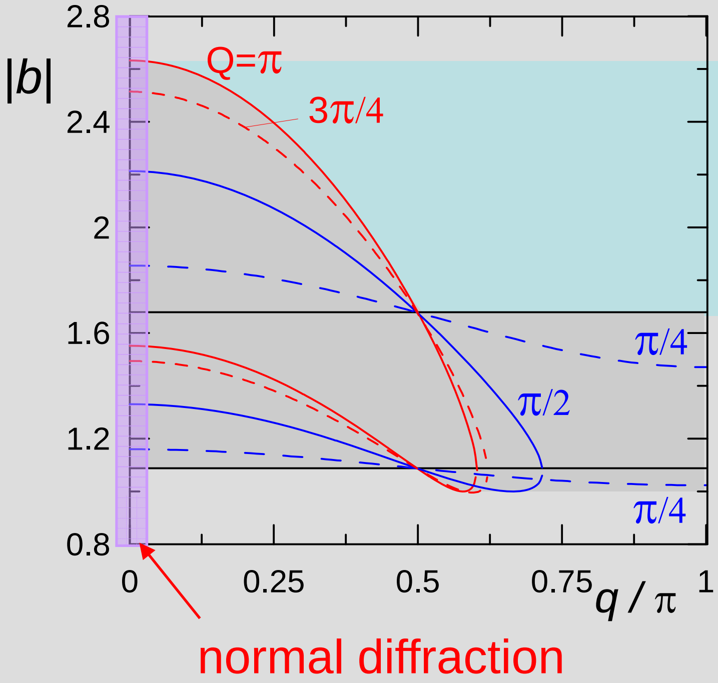
modulational instability $\rightarrow \Re(\lambda) > 0$

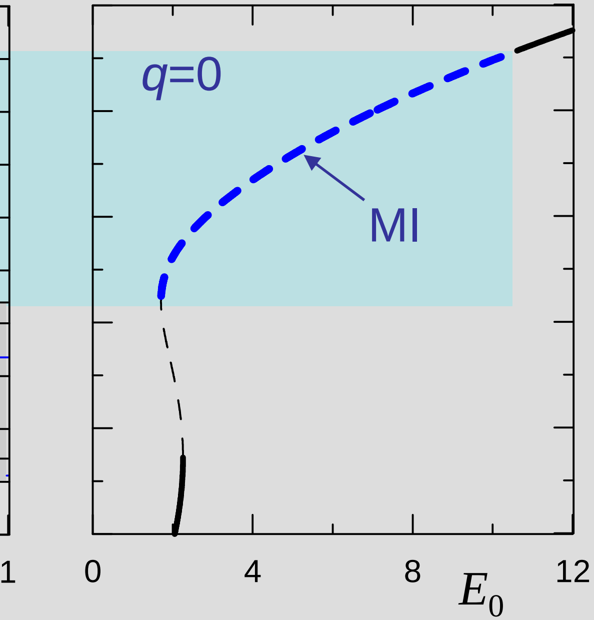
$$\lambda(Q,q) = -1 \pm \sqrt{\left(2C\left(\cos Q\cos q - 1\right) + \Delta + \gamma|b|^2\right)\left(2C\left(\cos Q\cos q - 1\right) + \Delta + 3\gamma|b|^2\right) - } - 2\,i\,C\sin Q\sin q$$

Modulational Instability of PW

$\gamma\Delta' < -\sqrt{3}$ - bistable case







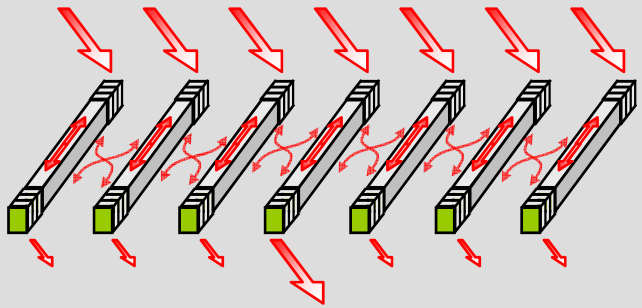
MI domains vs. modulation

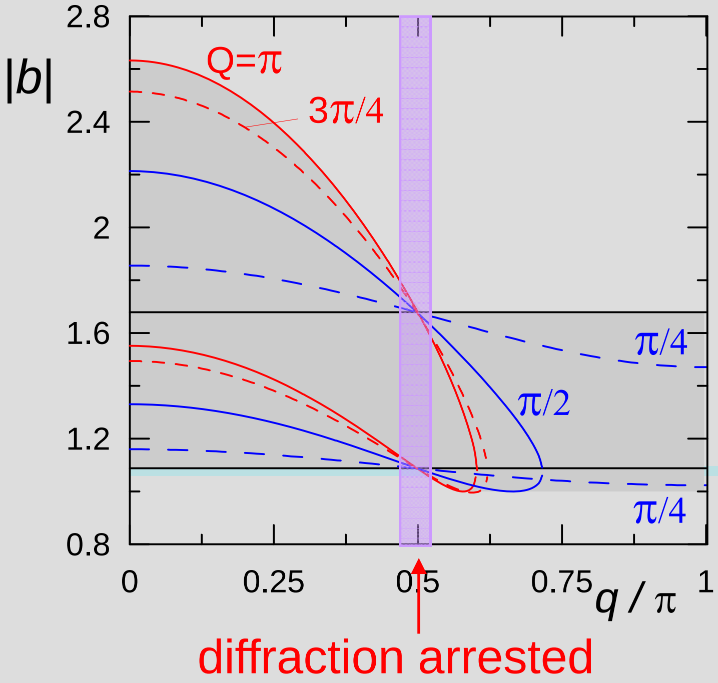
plane wave response

O. Egorov and F. Lederer, and Y. S. Kivshar, Opt. Exp. 15 (07) 4149

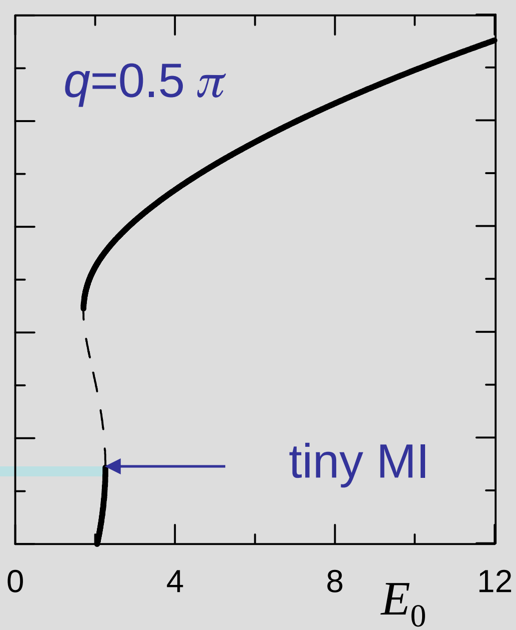
Modulational Instability of PW

$\gamma \Delta' < -\sqrt{3}$ - bistable case



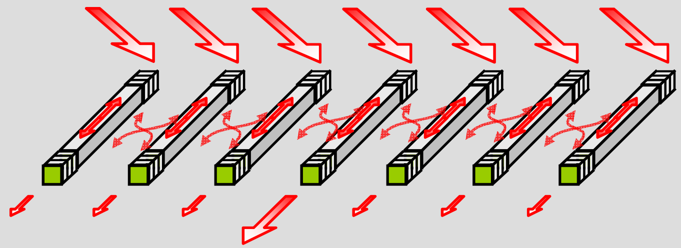


MI domains vs. modulation

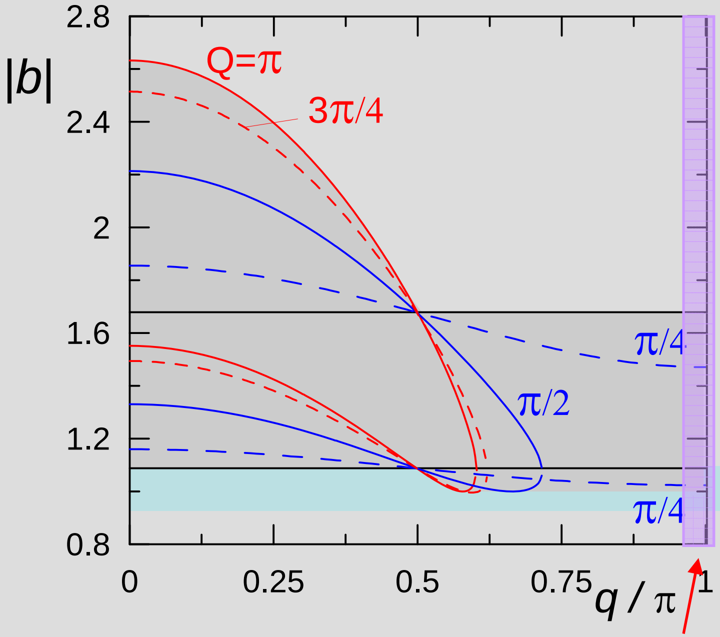


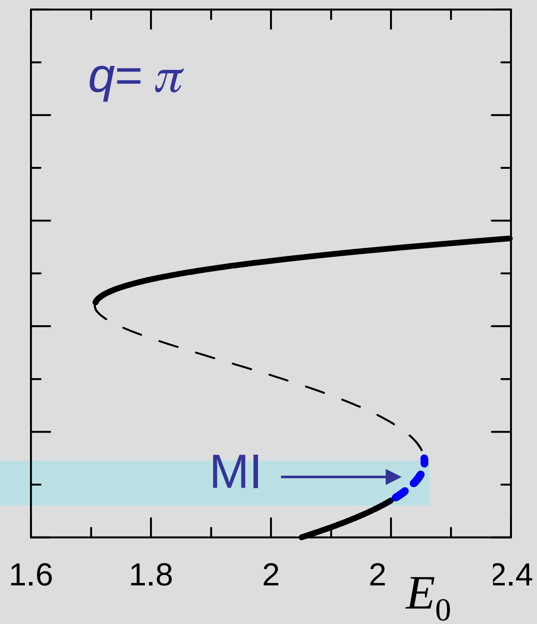
plane wave response

Modulational Instability of PW



$\gamma\,\Delta' < -\sqrt{3}$ - bistable case



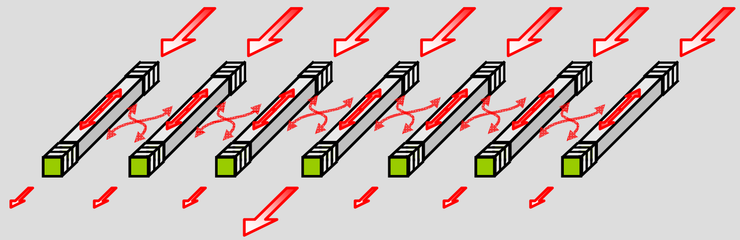


"anomalous" diffraction

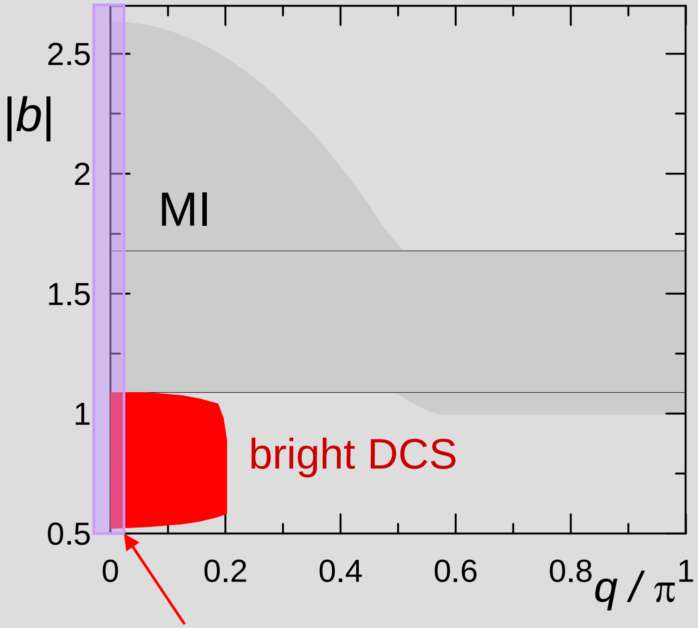
MI domains vs. modulation plane wave response

Soliton Solutions

Bright Solitons

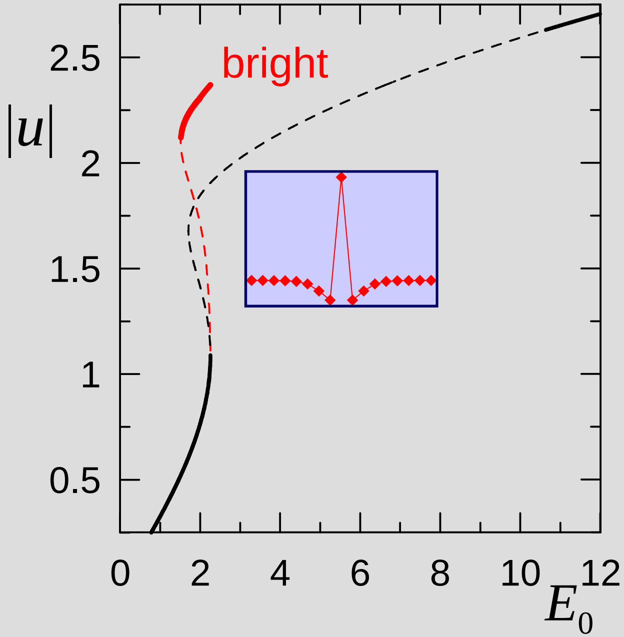


$q = 0$



DCS domains vs. inclination

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$



DCS domains vs. holding beam

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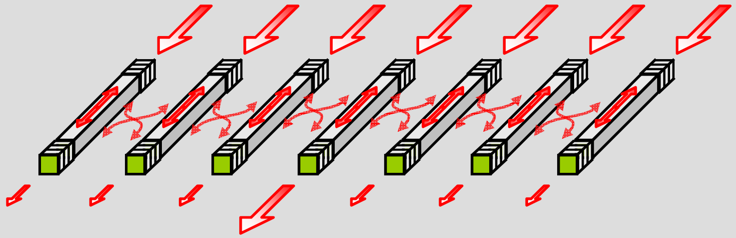
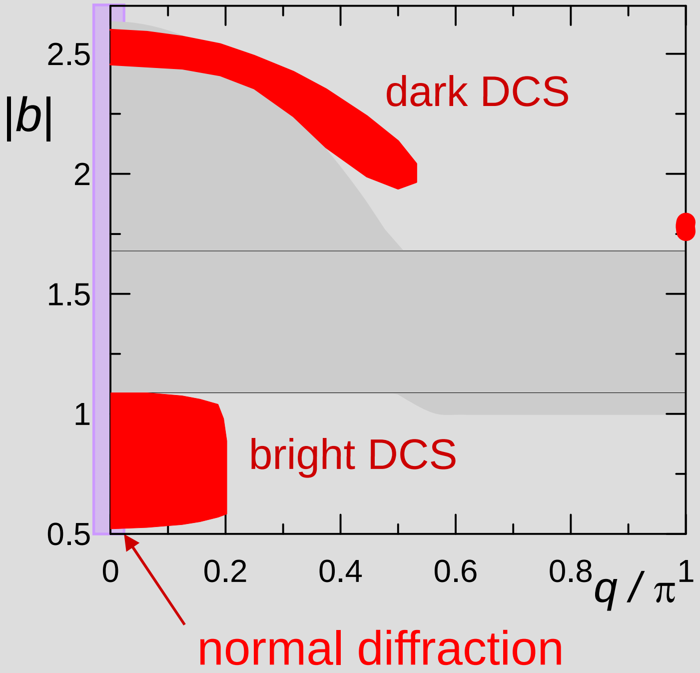
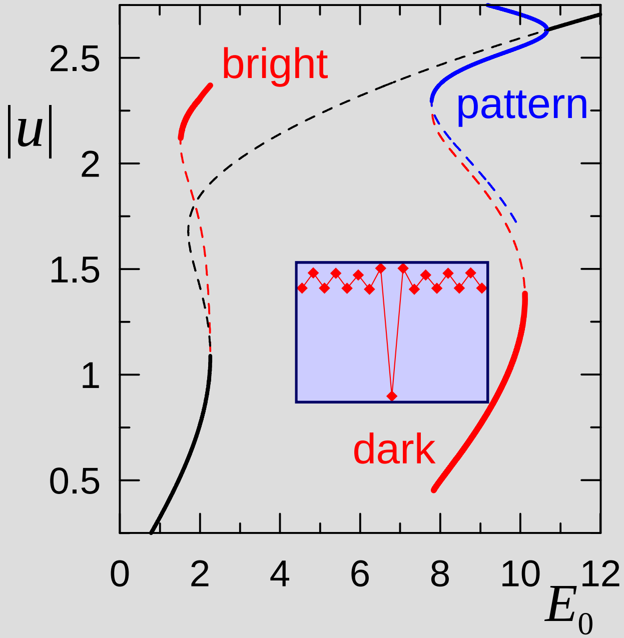
Dark Solitons on a Pattern



$q = 0$

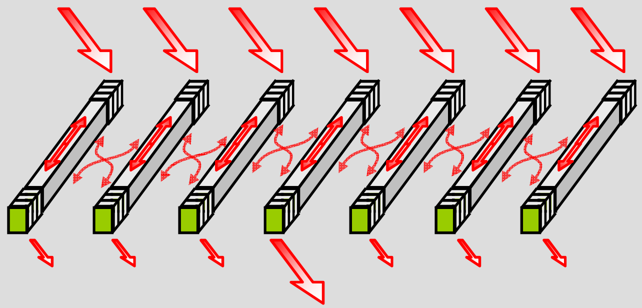




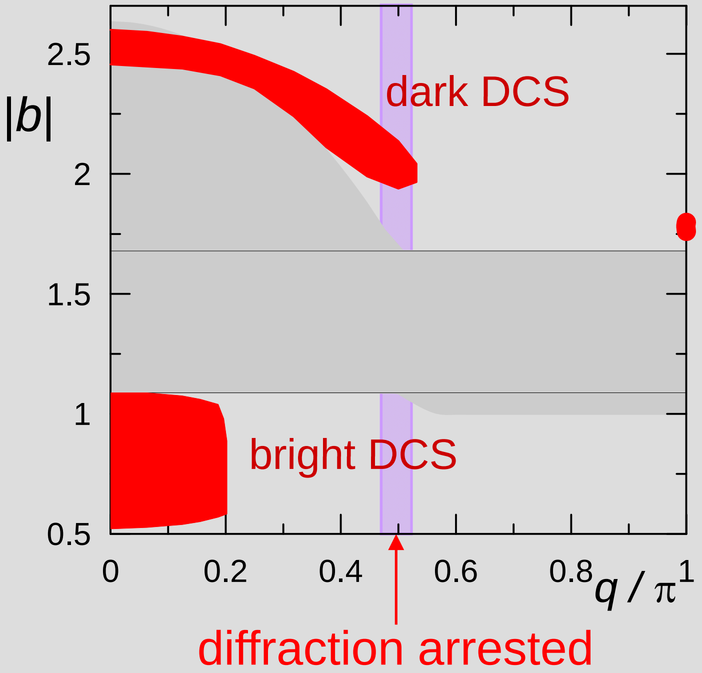
DCS domains vs. inclination DCS domains vs. holding beam

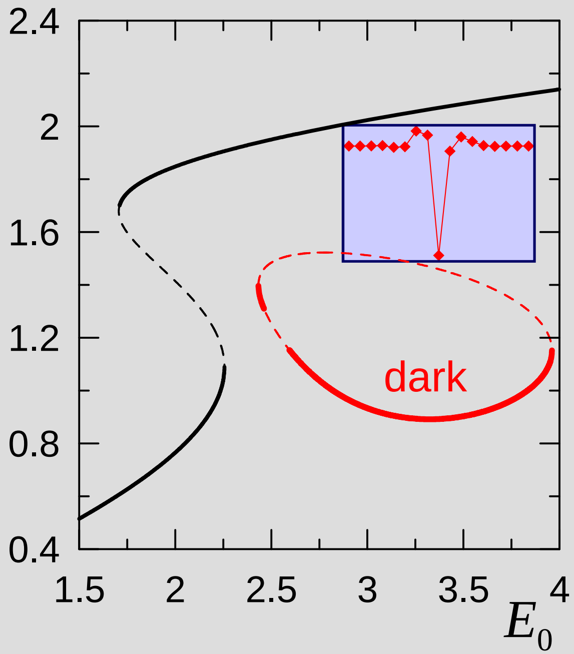
$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Dark Solitons



$q = \pi/2$



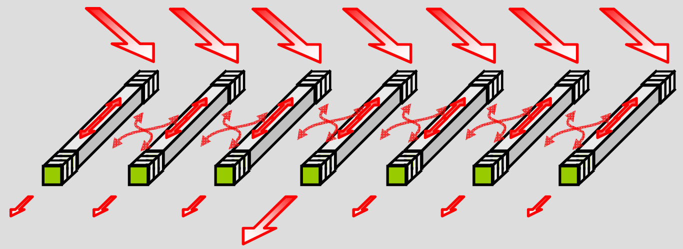


DCS domains vs. inclination

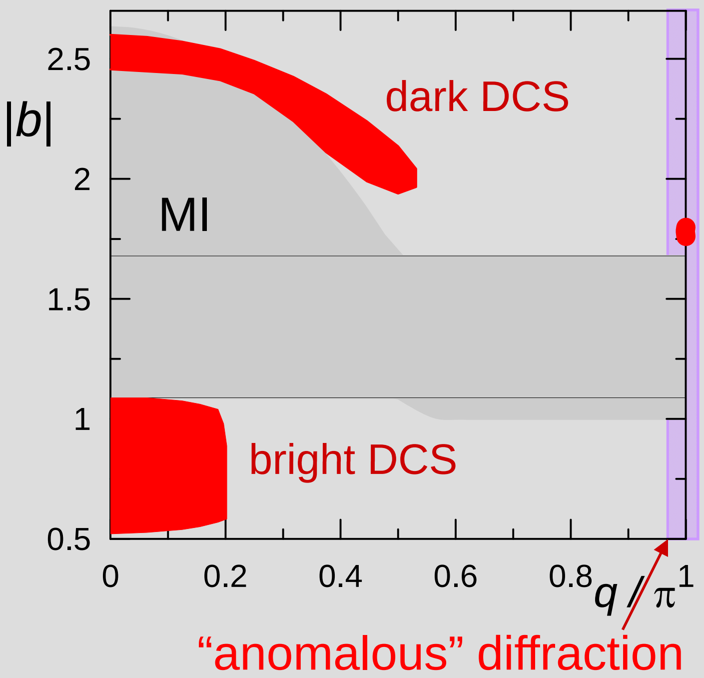
DCS domains vs. holding beam

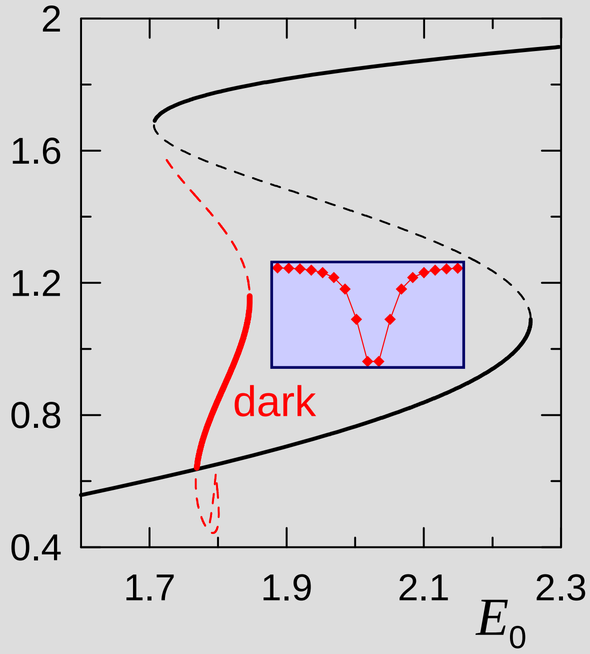
$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Dark Solitons



$q = \pi$





DCS domains vs. inclination DCS domains vs. holding beam

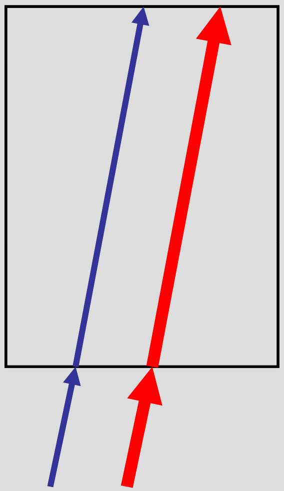
$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Outline

1. Introduction
2. Resting Discrete Cavity Solitons (DCS)
3. Mobility of Resting DCS vs. Moving DCS
4. Sub-diffractive DCS

Mobility of Discrete Solitons

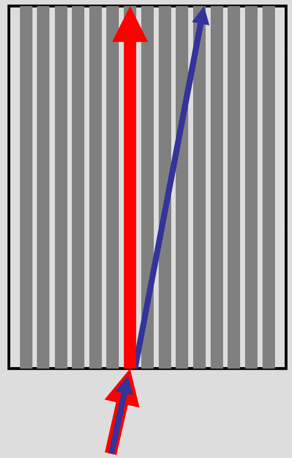
motion



continuous system – Galileian invariance

conservative systems

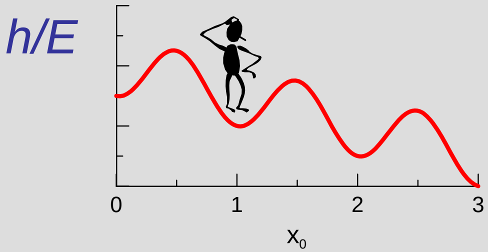
trapping



discrete system – no Galileian invariance

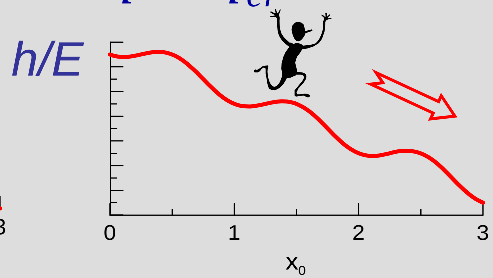
Peierls - Nabarro potential

$q < q_{cr}$



resting soliton

$q > q_{cr}$



moving soliton

Mobility - Continuous Model → Translational Mode

Linear stability analysis:

$$u(x) = U_s(x) + \delta u(x) e^{\lambda T}$$
$$v(x) = V_s(x) + \delta v(x) e^{\lambda T}$$

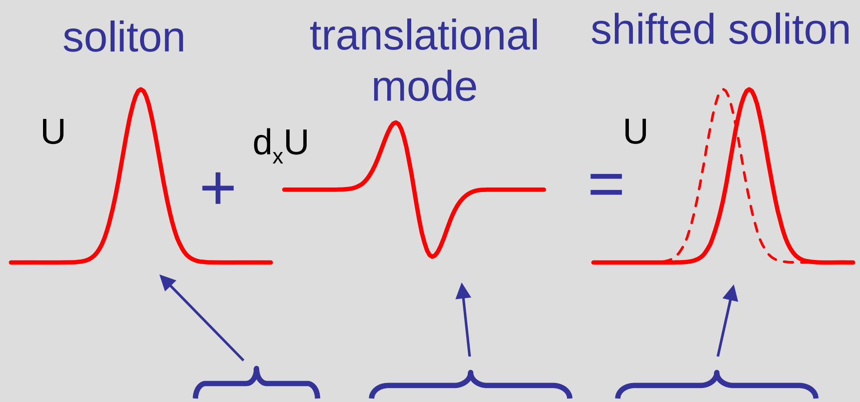
1) continuous case
(translational symmetry)

$$\left.\begin{aligned} \lambda &= 0 \\ \delta u(x) &= \partial_x U_s(x) \\ \delta v(x) &= \partial_x V_s(x) \end{aligned}\right\} \text{translational mode}$$

2) Discrete case
(loss of translational symmetry)

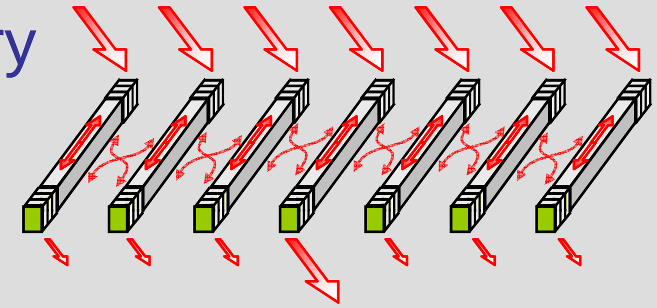
$$\lambda \neq 0$$
$$\delta u(x) \approx \partial_x U_s(x)$$
$$\delta v(x) \approx \partial_x V_s(x)$$

quasi-translational mode

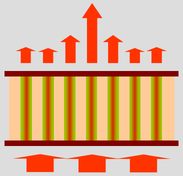


$$u(x) = U_s(x) + a\,\partial_x U_s(x) \approx U_s(x+a)$$
$$v(x) = V_s(x) + a\,\partial_x V_s(x) \approx V_s(x+a)$$

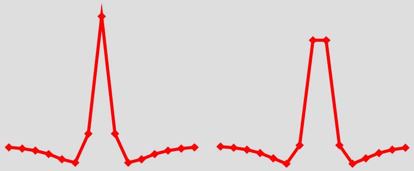
Motion of DCS - Perturbation Theory



1) Resting DCS



$q < q_{\mathrm{crit}}$



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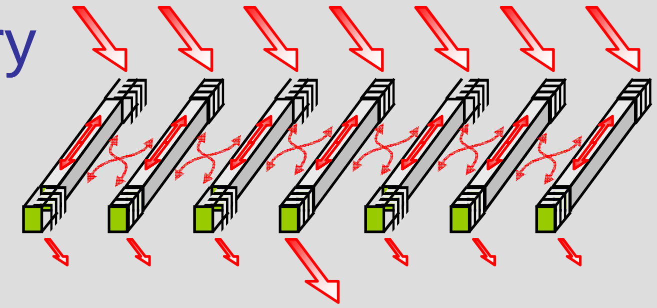
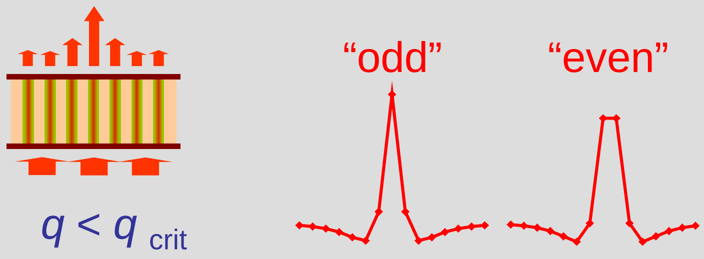
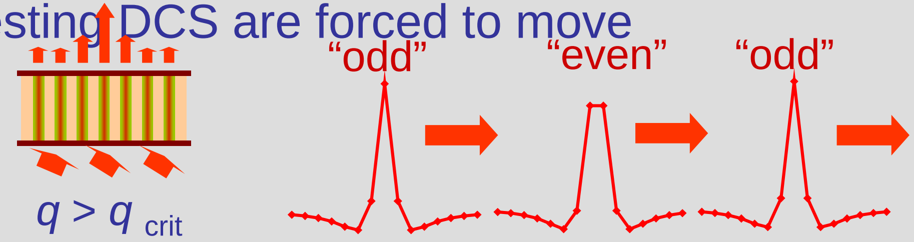
Motion of DCS - Perturbation Theory

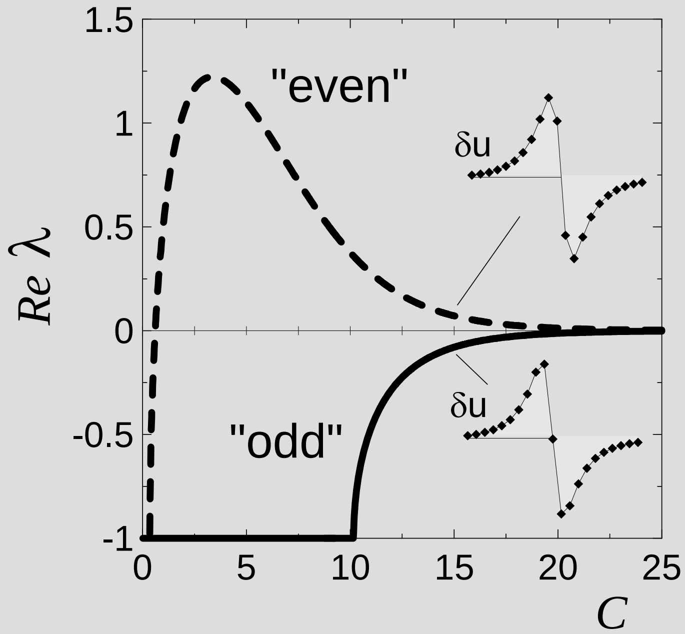


1) resting DCS



2) R
resting DCS are forced to move





eigenvalue of the “quasi-translational” mode

motion → transition between „odd“ and „even“ DCSs

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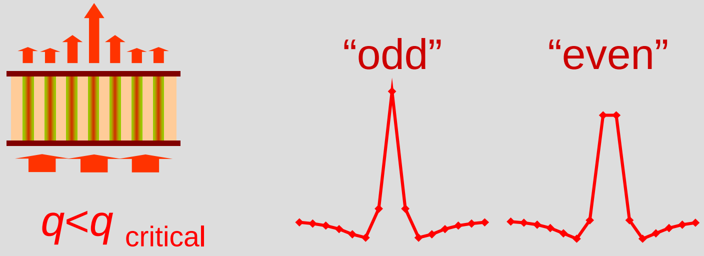
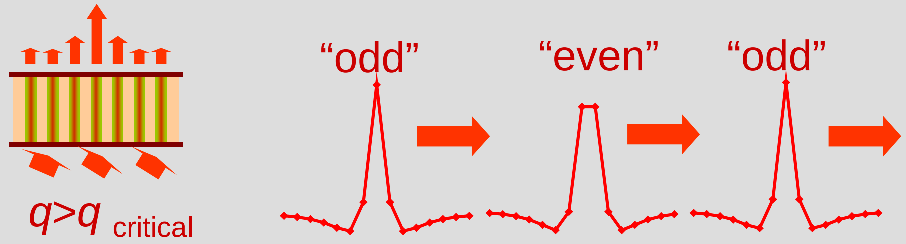
Motion of DCS - Perturbation Theory

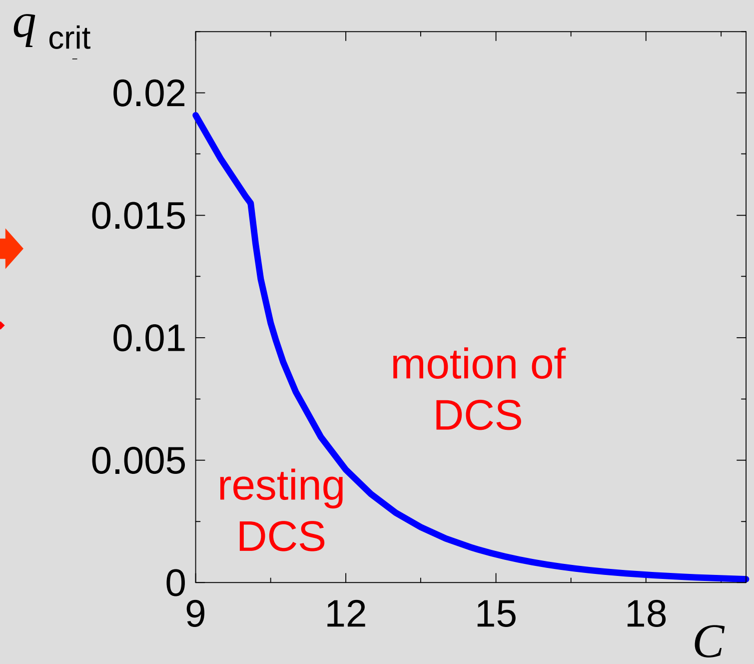
1) resting DCS



$$q_{\text{crit}} \approx \frac{\left|\Re\lambda(C)^{odd} - \Re\lambda(C)^{even}\right|}{C} \cdot K$$

2) motion of DCS





motion ⇒ transition between „odd“ and „even“ DCSs

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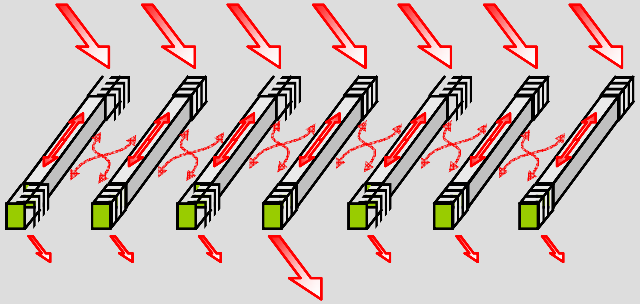
Moving DCS



Quasi-Continuous Approach

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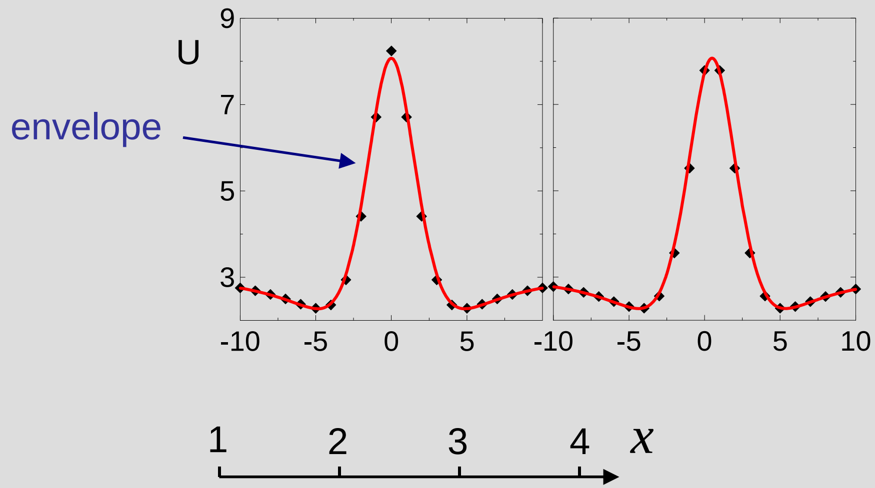
aim: - to establish an analytical model

idea: - to derive a quasi-continuous model for the DCS envelope by keeping properties of discrete diffraction (phase difference, coupling strength)

$\rightarrow$ soliton can be narrow



$$x_n = n\,h, \quad n = 1, 2, ..$$

Quasi-Continuous Approach

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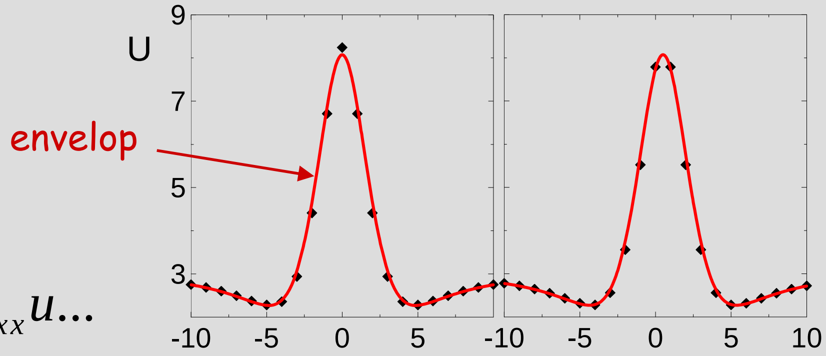
Continuous envelope:

$$u(x = nh)e^{iqn} \equiv u_n$$

Taylor expansion:

$$u_{n\pm1} \approx u_n \pm h\partial_x u + \tfrac{1}{2}h^2\,\partial_{xx}u \pm \tfrac{1}{6}h^3\,\partial_{xxx}u\ldots$$



Quasi-Continuous Approach

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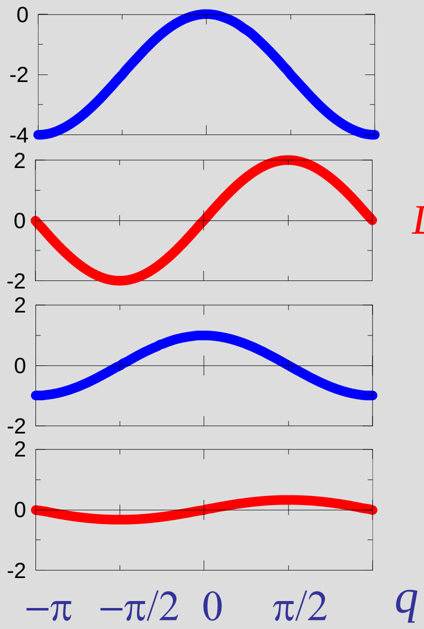

$D^{(0)} = 2C(\cos q - 1)$ - detuning

$D^{(1)} = 2C\, h \sin q$ - inclination, velocity

$D^{(2)} = Ch^2 \cos q$ - 2-order diffraction coefficient

$D^{(3)} = \left(Ch^3/3\right)\sin q$ - 3-order diffraction coefficient

$$i\frac{\partial u}{\partial T} + iD_u^{(1)}\frac{\partial u}{\partial x} + D_u^{(2)}\frac{\partial^2 u}{\partial x^2} + iD_u^{(3)}\frac{\partial^3 u}{\partial x^3} + \left[i + \left(\Delta_1 + D_u^{(0)}\right)\right]u + \gamma|u|^2 u = E_0$$

Quasi-Continuous Approach

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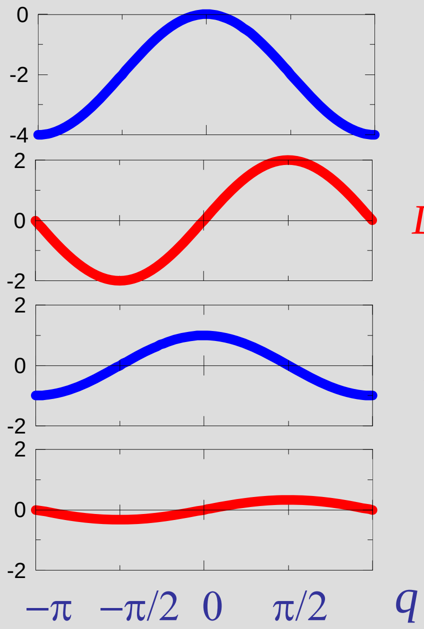


$D^{(0)} = 2C(\cos q - 1)$ - detuning

$D^{(1)} = 2C\, h \sin q$ - inclination, velocity

$D^{(2)} = Ch^2 \cos q$ - 2-order diffraction coefficient

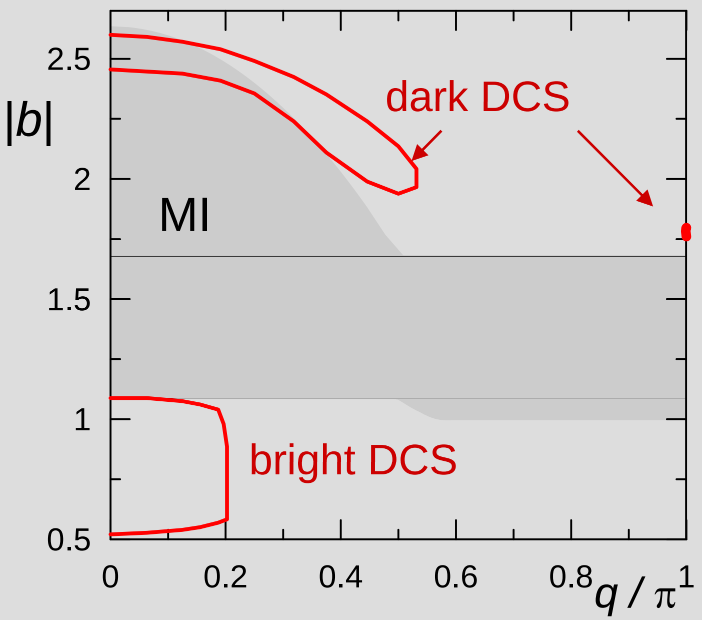
$D^{(3)} = (Ch^3/3)\sin q$ - 3-order diffraction coefficient

moving frame

$$i\frac{\partial u}{\partial T} + iD_u^{(1)}\frac{\partial u}{\partial x} + D_u^{(2)}\frac{\partial^2 u}{\partial x^2} + iD_u^{(3)}\frac{\partial^3 u}{\partial x^3} + \left[i + \left(\Delta_1 + D_u^{(0)}\right)\right]u + \gamma|u|^2 u = E_0$$

Moving Discrete Cavity Solitons

O. Egorov and F. Lederer,
and Y. S. Kivshar,
Opt. Exp. 15 (07) 4149



domains of resting DCSs $\Delta' < -3 \quad C = 1 \quad \gamma = 1$

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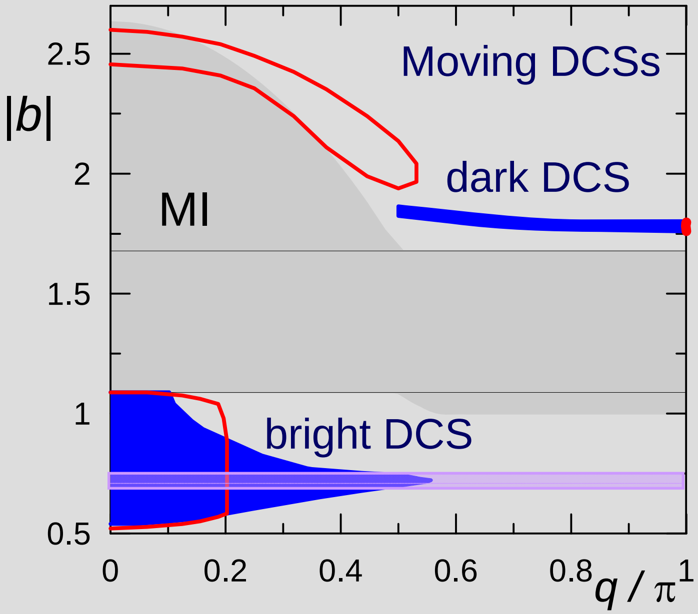

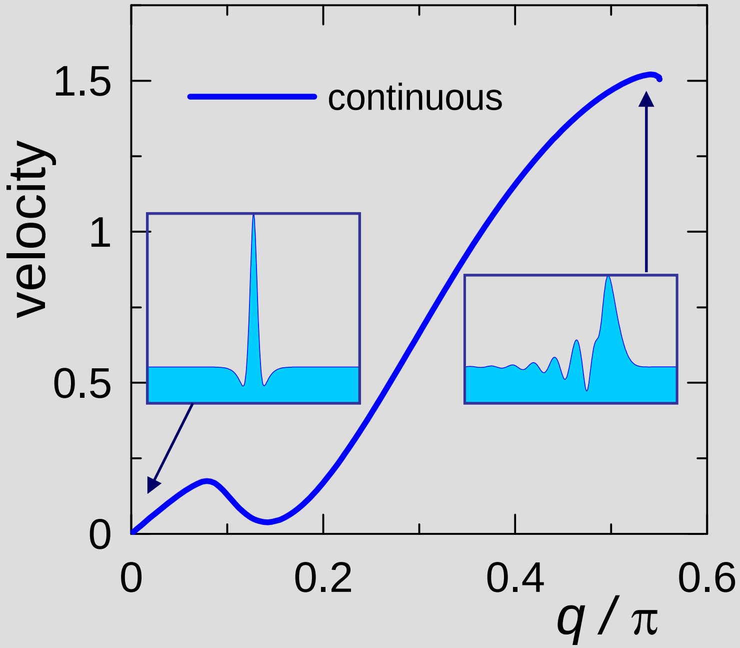

moving DCSs in the quasi-continuous model

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Moving Discrete Cavity Solitons

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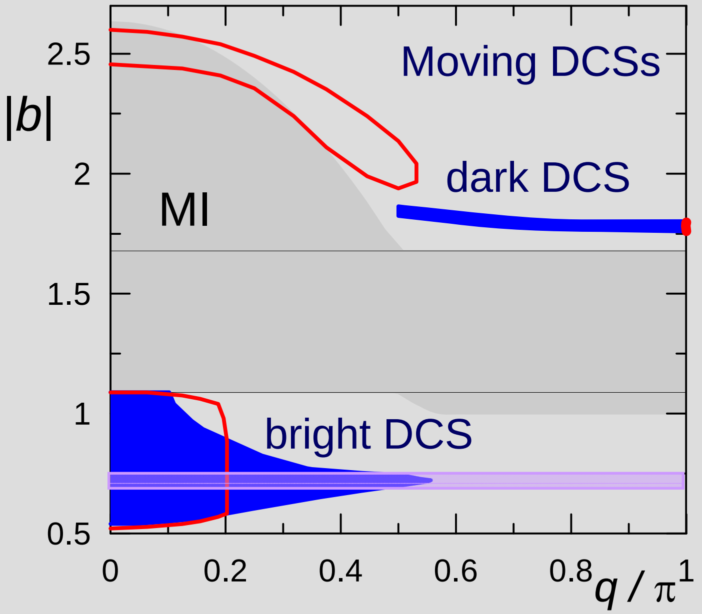

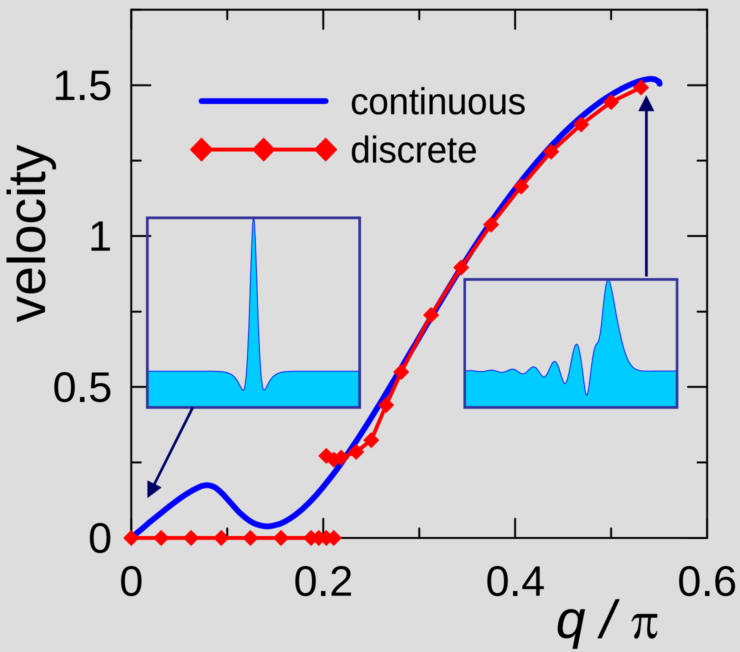

moving DCSs in the discrete model

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Moving Discrete Cavity Solitons

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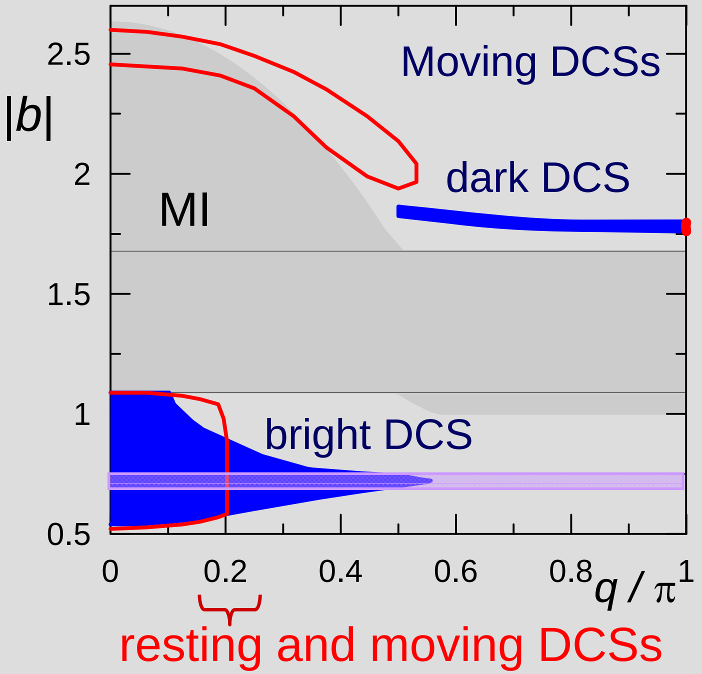

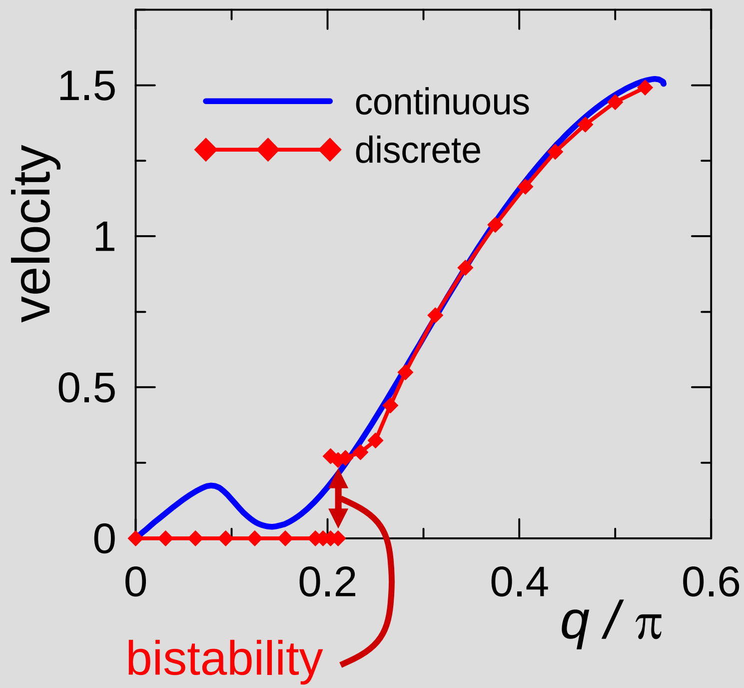

moving DCSs in the discrete model

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Moving Discrete Cavity Solitons

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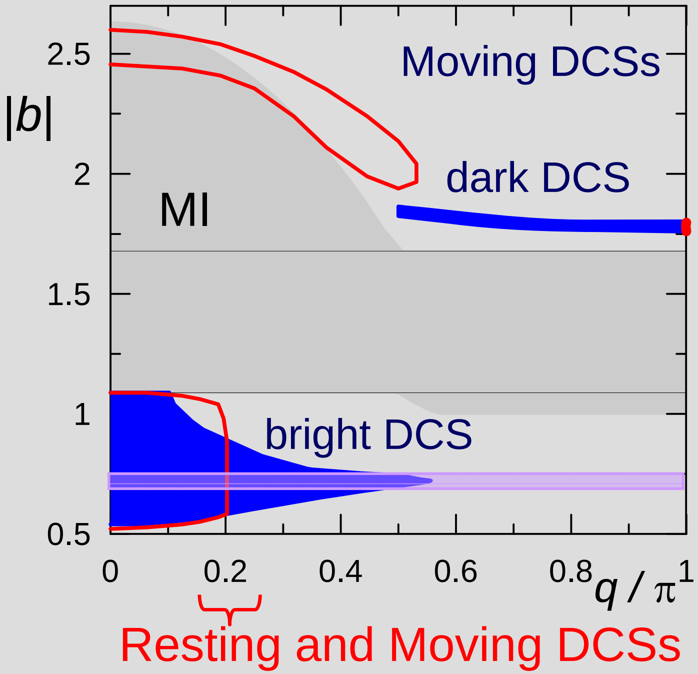
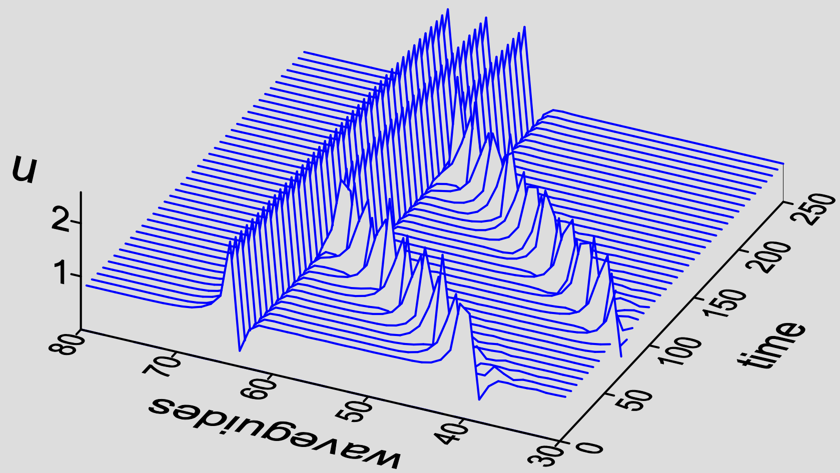
Opt. Exp. 15 (07) 4149



moving DCSs in the discrete model



collision between resting and moving discrete Cavity Solitons

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Moving Discrete Cavity Solitons

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and Y. S. Kivshar,
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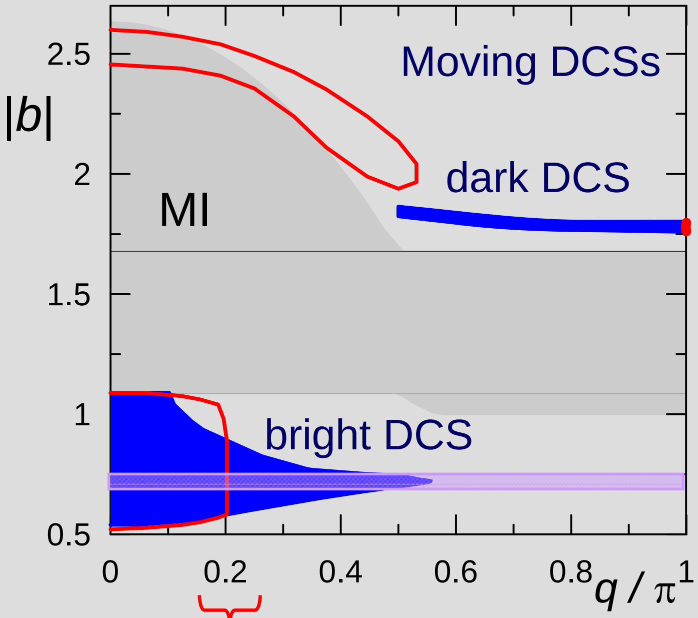
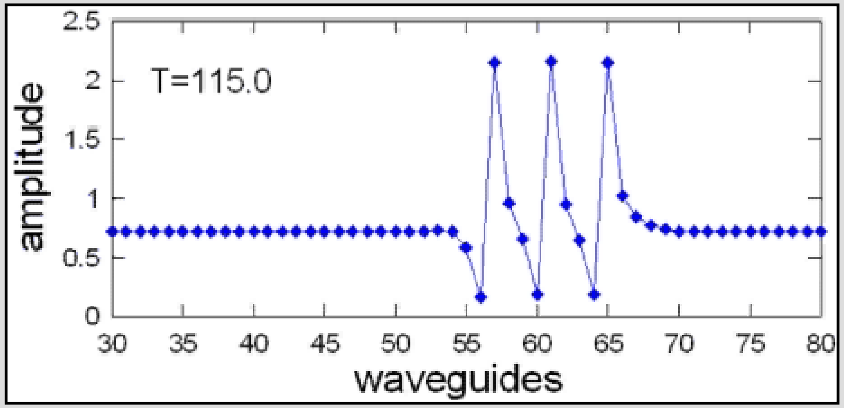


resting and moving DCSs

moving DCSs in the discrete model

soliton ménage à trois



Collision between resting and moving
Discrete Cavity Solitons

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Outline

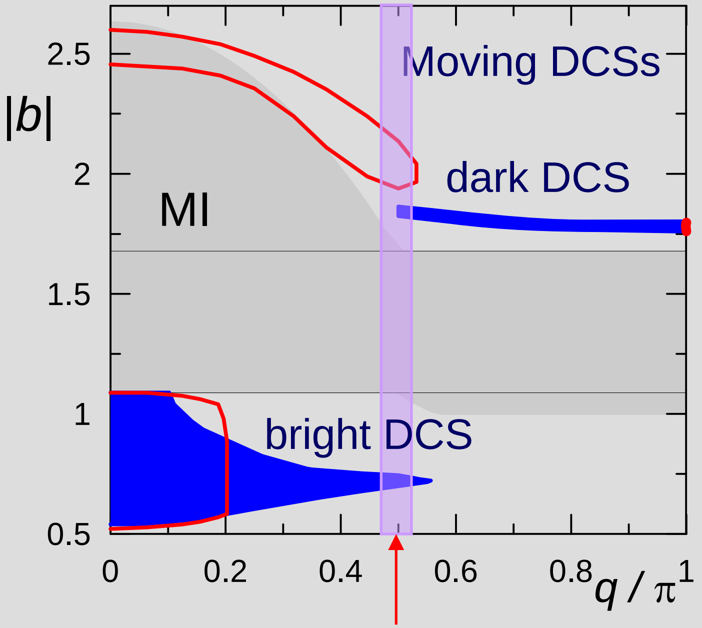
1. Introduction
2. Resting Discrete Cavity Solitons (DCS)
3. Mobility of Resting DCS vs. Moving DCS
4. Sub-diffractive DCS

Subdiffractive Solitons

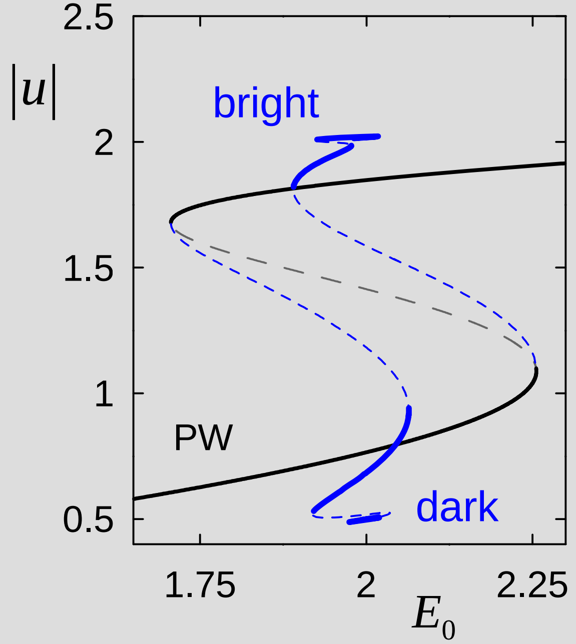
O. Egorov and F. Lederer, and K. Staliunas
Opt. Lett. (to appear August 1)

$$i\frac{\partial u}{\partial T}+iD_u^{(1)}\frac{\partial u}{\partial x}+D_u^{(2)}\frac{\partial^2 u}{\partial x^2}+iD_u^{(3)}\frac{\partial^3 u}{\partial x^3}+\left[i+\left(\Delta_1+D_u^{(0)}\right)\right]u+\gamma|u|^2u=E_0 \qquad D^{(2)}=Ch^2\cos q$$

$q = \pi/2$



diffraction arrested

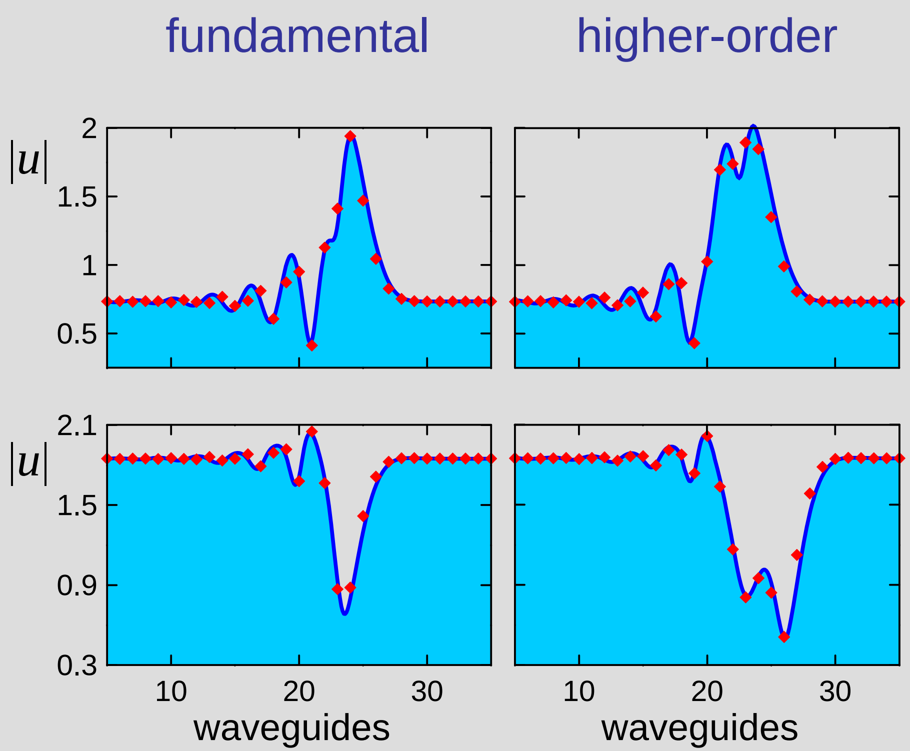


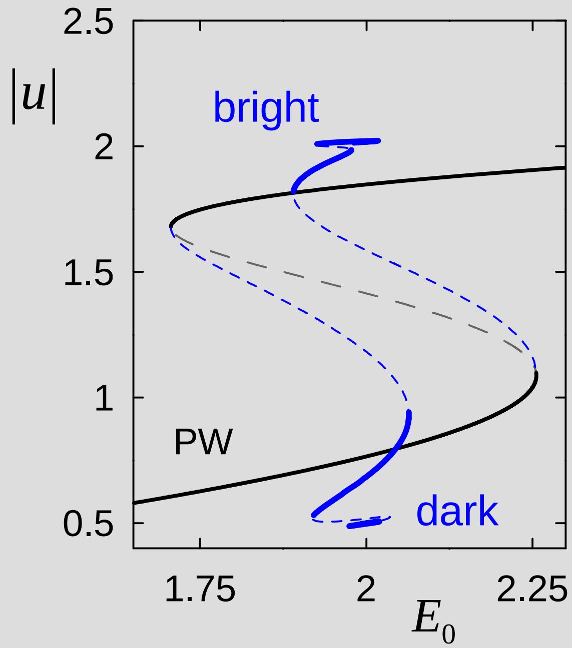
$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

moving DCSs in the discrete model

Subdiffractive Solitons

O. Egorov and F. Lederer, and K. Staliunas
Opt. Lett. (to appear August 1)





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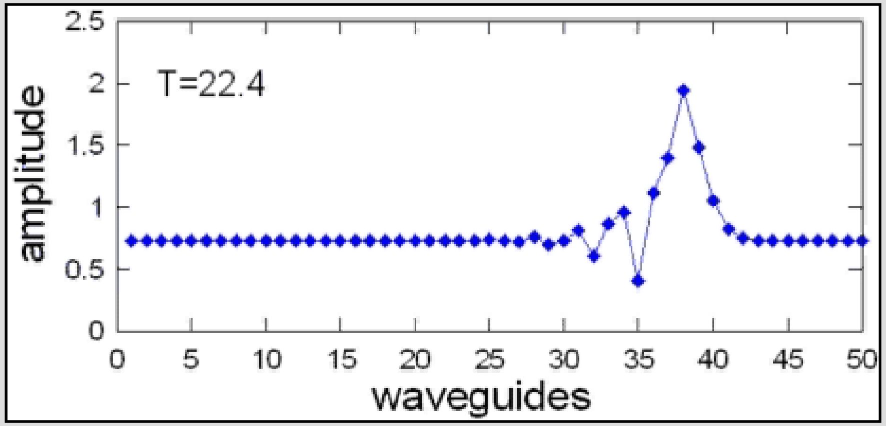

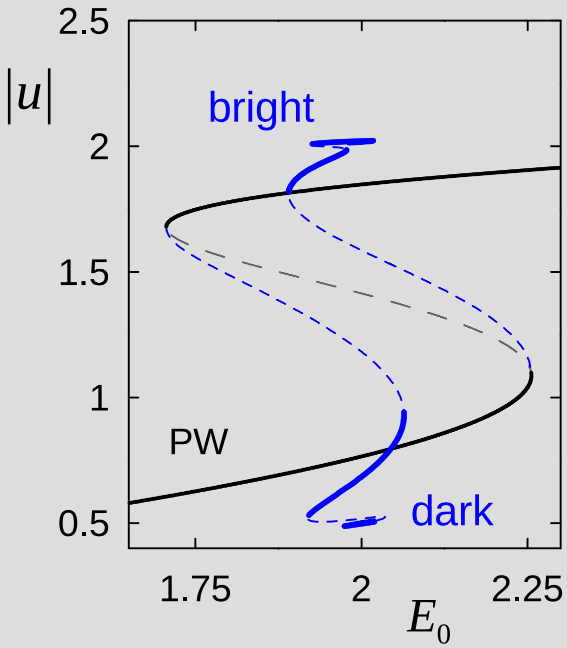

$\Delta' < -3 \quad C = 1 \quad \gamma = 1$

Conclusions

- MI and discrete cavity soliton formation depend strongly on the holding beam inclination
- beyond a critical inclination resting solitons start to move
- a quasi-continuous approach permits to identify moving DCSs
- subdiffractive DCS exist if second order diffraction dissapears